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MODERN MISSION DESIGN WITH THE HIGH INCLINED ORBIT FORMATION USING GRAVITY ASSISTS: MAIN METHODS

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Summary. An effective space exploration is impossible without gravity assists (GA) using. Their application relaxes the constraints imposed on the space mission scenarios by the characteristic velocity budgets being realized at the current stage of development of space technology. A significant change in the inclinations of operational spacecraft (SC) orbits in flight aimed at studying the inner heliosphere from out-of ecliptic positions (the ESA "Solar Orbiter" mission, Russian "Interheliozond") is needed to accomplish some prospective space missions. Low-cost tours for the high inclined orbit formation in the Solar system with use of gravity assists near its planets (Earth and Venus) with the full ephemeris using are considered. The limited dynamic possibilities of using gravity maneuvers require their repeated performance. Based on the formalization of the search for the GA- timetables with subsequent adaptive involvement of a large number of options, a high-precision algorithm for synthesizing chains of increasing gravity assists was built. Its use leads to a significant inclination change of the research SC's orbit without significant fuel consumption during a reasonable flight time.

1 INTRODUCTION

An effective space exploration is impossible without gravity assists (GA) using. Their application relaxes the constraints imposed on the space mission scenarios by the characteristic velocity budgets being realized at the current stage of development of space technology. A significant change in the inclinations of operational spacecraft (SC) orbits in flight aimed at studying the inner heliosphere from out of ecliptic positions (the ESA "Solar Orbiter" mission, the Russian "Interheliozond" project, etc.) is needed to accomplish some prospective space missions. Low-cost tours for the high inclined orbit formation in the Solar system with use of gravity assists near its planets (Earth and Venus) with the full ephemeris using are considered. The limited dynamic possibilities of using gravity maneuvers require their repeated performance. Relevance of regular creation of optimum scenarios — sequences of cranking passing of celestial bodies and solution of conditions of their execution is obvious. The technology for synthesizing such scenarios is complicated by the necessity of their 3D design with allowance made for precise ephemeris models. The formalism is based on two basic factors for designing high-inclination orbits. The first factor is geometric restrictions on the maximum possible inclination of the SC's orbit, which is achievable depending on the relative value of the excess vector of the SC's hyperbolic velocity (the

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asymptotic velocity of the SC relative to the planet) compared to the average orbital velocity of the planet for any flight sequence. The second factor is dynamic restrictions on the maximum angle of rotation of the asymptotic velocity vector of the SC during a single gravity assist. Dynamic restrictions also depend on the value of the asymptotic velocity of the SC and the gravitational parameters of the planet. A joint analysis of the factors presented makes it possible to draw a conclusion about the dynamic nature of the planned space mission, which, however, will require further clarification. Based on the formalization of the search for the such scenarios with subsequent adaptive involvement of a large number of options, a high-precision algorithm for synthesizing chains of increasing gravity assists was built. Its use leads to a significant change in the inclination of the research SC's orbit without significant fuel consumption during a reasonable flight time.

In previous works [1-3], the authors conducted a comparative analysis of various modern astrodynamics developments [4-8], affecting the 3D spatial implementation of gravity assists, in order to clarify the possibility of their application, taking into account the exact ephemeris. In [3] a refined analytical formula of the inclination changes of the SC orbit were obtained for one-pass 3D GA obtained and the results of calculations by using the parameters changes of the orbital inclination of the SC relative the Solar system planets and their satellites. In this paper, the main focus is on the development of algorithms for the synthesis of multi-pass GA chains, the use of which leads to a significant increase in the inclination of the SC orbit above the ecliptic plane.

In this paper, we generalize the formulas of [1] for the inclination of the SC orbit and its changes during gravitational maneuvers near the planet to the overall case of elliptical orbits of not only the SC, but also the planet. The geometric interpretation of the obtained formulas is presented. Expressions are found for the coordinates of the inclination pole on the invariant sphere of the asymptotic velocity of the SC. The procedure for GA chains reaching the inclination pole to achieve the geometrically acceptable maximum inclination of the SC orbit during multi-pass gravitational maneuvers is analytically studied.

2 GEOMETRIC RESTRICTIONS OCCURRED DURING GRAVITY ASSISTS PERFORMING

Using the results of [1, 2], we introduce spherical coordinates to describe the invariant sphere of the position of the ends of the CS's asymptotic velocity vector $\mathbf{V}_\infty$ of the during gravity assists: radius V_∞ and angles ρ, σ. The angle ρ is the angle between the vector $\mathbf{V}_{\infty,out}$ obtained after the gravity assist and its projection on the orbital plane, and the angle σ is the angle between this projection and the orbital velocity vector of the planet $\mathbf{V}_{pl}$.

The restrictions on changing the inclination i of the SC's orbit when performing GA with a selected planet ("solo" GA) can be interpreted as geometric and dynamic [1, 2].

Geometric restrictions define the maximum value of i for any number of solo GA, which for the case (1) is given by the dimensionless asymptotic velocity of the KA v_∞ [1-3, 8-12]:

$$\sin i_{\max} = v_\infty , \tag{1}$$

$$v_\infty = V_\infty / V_{pl} . \tag{2}$$

It is easy to see that from (1) and (2) follows a restriction for the maximum possible inclination:

$$i_{\max} < \pi/2\,. \tag{3}$$

3 GRAVITY ASSISTS DYNAMIC RESTRICTIONS

Dynamic restrictions of the SC's orbit inclination changing are determined by the magnitude of the planet's gravitational field and the minimum allowable flyby distance near it. For the rotation angle φ of the SC's asymptotic velocity vector after a single-pass GA, is valid the formula [4]:

$$\sin\frac{\varphi}{2} = \frac{\mu}{\mu + r_\pi V_\infty^2} \tag{4}$$

where μ – gravitational parameter of the flyby body, r_π – the distance of the pericenter of the SC'’s flyby hyperbola, which cannot be less than the radius of the partner planet R_{pl}. The position of the "inclination pole" - the extremum point $\mathrm{T_{Pole}} : i = i_{\max}$ on the V_∞-sphere is schematically shown in Fig. 1. the Sequence of any solo GA in order to increase the inclination of the SC's orbit (we will call them “increasing chains”) should be as close as possible to the point $\mathrm{T_{Pole}}$.

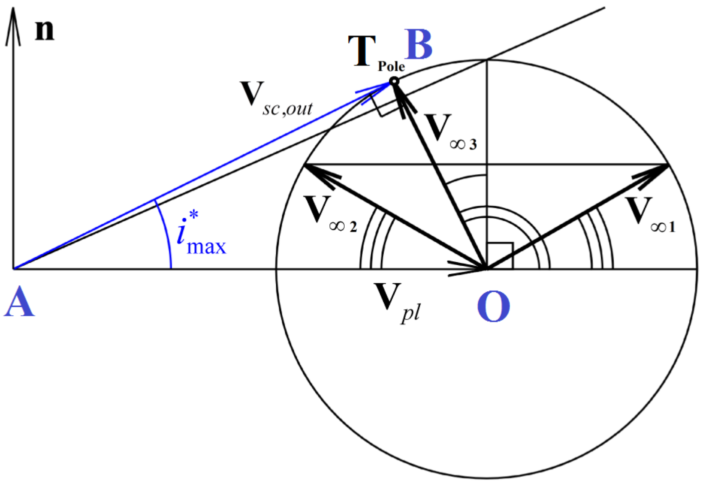

Fig. 1. The inclination pole $\mathrm{T_{Pole}}$ - the inclination extremum point location on the V_∞ - sphere in case $\sin\sigma = 0$

The overall meaning of dynamic constraints for GA is that the end of the output vector $\mathbf{V}_{\infty,out}$ for a single-pass GA does not go beyond the spherical region ("spherical cap") $S_{c\varphi}$. This region is the intersection of a sphere and a solid angle formed by a cone with a solution angle of 2φ, the axis of which is the vector of the input (before GA) SC's asymptotic velocity $\mathbf{V}_{\infty,in}$ (Fig. 2). The base of the spherical cup is obviously a circle K_φ of radius $r_\infty = V_\infty \sin\varphi$.

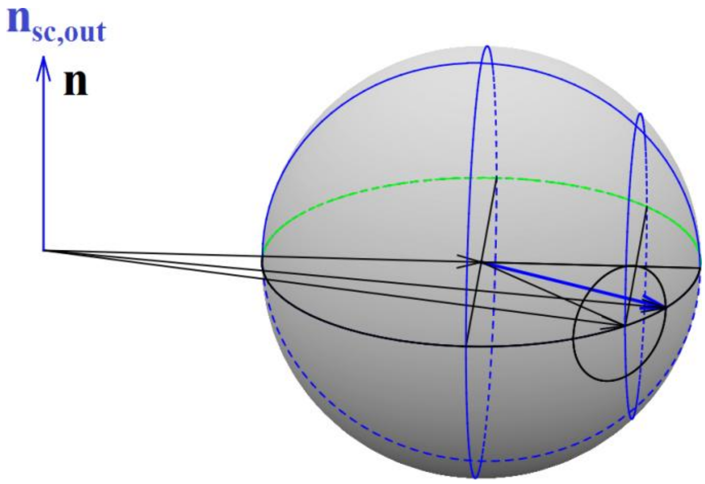

Fig. 2. The end of the output vector $\mathbf{V}_{\infty,\text{out}}$ for the single-pass GA does not extend beyond the spherical region $S_{c\varphi}$ ("spherical cap)

4 OVEERALL CASE OF THE PLANET'S ELLIPTIC ORBIT

Let's obtain a generalization of the results [1] for the case of elliptical orbits of the planet and SC. We then use the formula for the tangent of the desired angle of inclination i between two planes with normals $\mathbf{n},\mathbf{n}_{sc}$ under the condition (3):

$$\text{tg}\, i = \frac{|\mathbf{n}\times\mathbf{n}_{\text{sc}}|}{\mathbf{n}\cdot\mathbf{n}_{\text{sc}}} \tag{5}$$

Let γ is the trajectory angle (the angle between the velocity vector of a planet and the" virtual " vector of the circular orbital velocity of that planet at the same point). For a circular orbit [1] γ=0. We introduce the right triple of Cartesian coordinates (X,Y,Z) so that the axis X it is directed along the velocity vector of the planet, and Y is orthogonal to its orbital plane. Then $\mathbf{V}_{pl} = (V_{pl},0,0)$, $\mathbf{r}_{pl} = (r_{pl}\sin\gamma, r_{pl}\cos\gamma, 0)$, and

$$\mathbf{n} = \mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}} = (0,\ 0,\ -r_{\text{pl}}V_{\text{pl}}\cos\gamma),$$

$$\mathbf{n}_{\text{sc}} = \mathbf{r}_{\text{pl}}\times\left(\mathbf{V}_{\text{pl}}+\mathbf{V}_{\infty}\right),$$

$$\mathbf{n}\times\mathbf{n}_{\text{sc}} = \left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\times\left(\mathbf{r}_{\text{pl}}\times\left(\mathbf{V}_{\text{pl}}+\mathbf{V}_{\infty}\right)\right) =$$

$$= \left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\times\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)+\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\times\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\infty}\right) = \left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\times\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\infty}\right).$$

Let's use the identity: $(\mathbf{a}\times\mathbf{b})\times(\mathbf{a}\times\mathbf{c}) = \mathbf{a}(\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}))$, , which in this case will mean:

$$\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\times\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\infty}\right) = \mathbf{r}_{\text{pl}}\left(\mathbf{r}_{\text{pl}}\cdot\left(\mathbf{V}_{\text{pl}}\times\mathbf{V}_{\infty}\right)\right).$$

Writing $\mathbf{V}_{\infty}$ as:

$$\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\times\left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\infty}\right) = \mathbf{r}_{\text{pl}}\left(\mathbf{r}_{\text{pl}}\cdot\left(\mathbf{V}_{\text{pl}}\times\mathbf{V}_{\infty}\right)\right) \tag{6}$$

we find

$$\mathbf{V}_{\text{pl}} \times \mathbf{V}_{\infty} = (0,\ -V_{\text{pl}}V_{\infty}\sin\rho,\ V_{\text{pl}}V_{\infty}\cos\rho\sin\sigma) = (0,\ -V_{\text{pl}}V_{\infty z},\ V_{\text{pl}}V_{\infty y}) \quad (7)$$

and

$$\mathbf{n}\times\mathbf{n}_{\text{sc}} = \mathbf{r}_{\text{pl}}\left(\mathbf{r}_{\text{pl}}\cdot\left(\mathbf{V}_{\text{pl}}\times\mathbf{V}_{\infty}\right)\right) = -\mathbf{r}_{\text{pl}}\cdot r_{\text{pl}}V_{\text{pl}}V_{\infty z}\cos\gamma \quad (8)$$

This implies an important statement [1].

Statement. The SC's orbit normal vector $\mathbf{n}_{sc}$ after GA will always be orthogonal the radius vector of the planet $\mathbf{r}_{pl}$, that is, for any GA, $\mathbf{n}_{sc}$ it always remains in the plane that is orthogonal $\mathbf{r}_{pl}$.

Expression (7) can be represented in coordinate form. We calculate for such a representation $\mathbf{n}\cdot\mathbf{n}_{\text{sc}}$:

$$\begin{aligned}\mathbf{n}\cdot\mathbf{n}_{\text{sc}} &= \left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\cdot\left(\mathbf{r}_{\text{pl}}\times\left(\mathbf{V}_{\text{pl}}+\mathbf{V}_{\infty}\right)\right) = \left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)^2 + \left(\mathbf{r}_{\text{pl}}\times\mathbf{V}_{\text{pl}}\right)\cdot\left(\mathbf{r}_{\text{pl}}\cdot\mathbf{V}_{\infty}\right) = \\ &= r_{\text{pl}}^2\cdot V_{\text{pl}}^2\cdot\cos^2\gamma + r_{\text{pl}}^2\left(\mathbf{V}_{\text{pl}},\mathbf{V}_{\infty}\right) - \left(\mathbf{r}_{\text{pl}}\cdot\mathbf{V}_{\text{pl}}\right)\cdot\left(\mathbf{r}_{\text{pl}}\cdot\mathbf{V}_{\infty}\right)\end{aligned} \quad (9)$$

Using relations

$$\mathbf{V}_{\text{pl}}\cdot\mathbf{V}_{\infty} = V_{\text{pl}}V_{\infty x},\ \mathbf{r}_{\text{pl}}\cdot\mathbf{V}_{\text{pl}} = r_{\text{pl}}V_{\text{pl}}\sin\gamma,$$
$$\mathbf{r}_{\text{pl}}\cdot\mathbf{V}_{\infty} = r_{\text{pl}}V_{\infty x}\sin\gamma + r_{\text{pl}}V_{\infty y}\cos\gamma \quad (10)$$

we can find:

$$\begin{aligned}\mathbf{n}\cdot\mathbf{n}_{\text{sc}} &= r_{\text{pl}}^2\cdot V_{\text{pl}}^2\cdot\cos^2\gamma + r_{\text{pl}}^2V_{\text{pl}}V_{\infty x} - r_{\text{pl}}^2V_{\text{pl}}\sin\gamma\left(V_{\infty x}\sin\gamma + V_{\infty y}\cos\gamma\right) = \\ &= r_{\text{pl}}^2\cdot V_{\text{pl}}^2\cdot\cos^2\gamma + r_{\text{pl}}^2V_{\text{pl}}V_{\infty x}\cos^2\gamma - r_{\text{pl}}^2V_{\text{pl}}V_{\infty y}\sin\gamma\cos\gamma.\end{aligned} \quad (11)$$

As a result, we get the formula for the tangent of the angle of inclination:

$$\text{tg}\, i = \frac{V_{\infty z}}{V_{\text{pl}}\cos\gamma + V_{\infty x}\cos\gamma - V_{\infty y}\sin\gamma}, \quad (12)$$

$$\text{tg}\, i = \frac{V_{\infty}\sin\rho}{V_{\text{pl}}\cos\gamma + V_{\infty}\cos\rho\cos(\gamma+\sigma)}. \quad (13)$$

The obtained formula (13) can be considered as a functional relation $tg\ i(\rho,\sigma)$.

Note that the parameter γ**,** as a parameter of the planet's orbit, it does not depend on the point of GA. It follows from (13) and (1) that the maximum of the function $tg\ i$ is reached at a certain pole of inclination on the V_{∞} - sphere $\sigma = \sigma^*, \rho = \rho^*$, for which is properly:

$$\cos(\gamma+\sigma^*) = -1,\ \sigma^* = \pi - \gamma, \quad (14)$$
$$\cos\rho^* = \frac{V_{\infty}/V_{pl}}{\cos\gamma} = \frac{v_{\infty}}{\cos\gamma} = \sin i_{\max}.$$

For the case of a circular orbit of the planet, the relation will be fulfilled $\sigma^* = \pi$.

5 BASIC ANGLES OF GRAVITY ASSISTS MANEUVERS FOR PLANETS

We compare the dependence of the maximum angle of rotation of the asymptotic velocity vector of the SC $\varphi_{\max}$, substituting the condition in (4) $r_{\pi} = R_{pl}$, and geometrically acceptable inclination of the formed orbit of the spacecraft (2) for the planets of the Solar system.

Comment. Comparison of (2) and Fig. 1 [4], formula (1.2.10), shows that in [4] the value of $V_{sc,out}$ approximately replaced in the denominator by V_{pl}, so that:

$$\sin i \approx \frac{V_\infty}{V_{\mathrm{pl}}} \sin\varphi_{\max} = v_\infty \sin\varphi_{\max} . \tag{15}$$

Expression (15) in some cases , for not very large values of v_∞, can serve as a satisfactory approximation of $\sin i$ [1, 2, 3].

The graphs $\varphi_{\max}$ relative V_∞ for the terrestrial planets and Jupiter are shown in Fig. 3. They show that the maximum rotation angles are reached at near-zero values V_∞. However, the value of $i_{\max}$ in this case, according to (4), it is close to zero.

GA "efficiency" appears only when increasing V_∞ to the values that provide the required value for a space mission $i_{\max}$, but at the same time the value $\varphi_{\max}$, which is demonstrated in this graph. The bold line denotes the model value of the demanded design inclination angle $i_{\max} = \pi/6$. The vertical, lowered from the point of its intersection with the graph of the function of the maximum inclination of the planet, shows the corresponding value of the rotation angle $\varphi_{\max}$ of the SC's asymptotic velocity vector on one single GA.

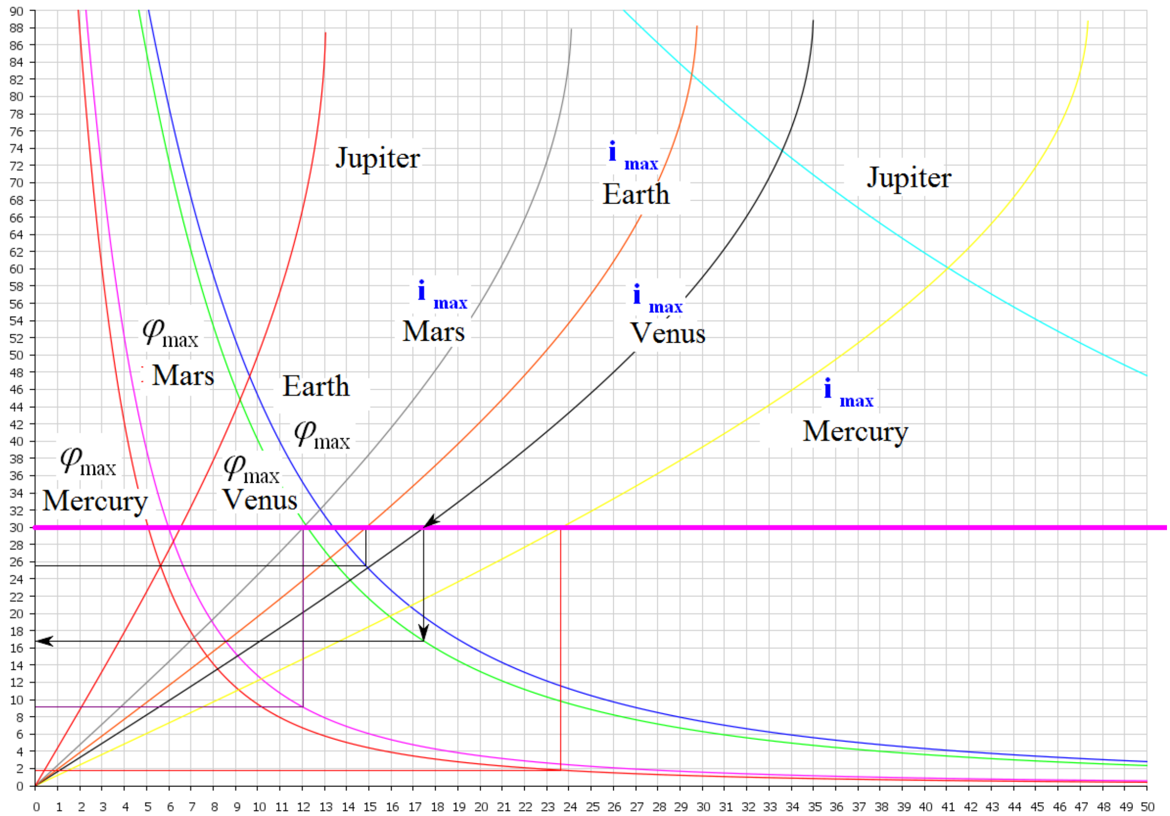

Fig. 3. The dependence of $\varphi_{\max}$ and $i_{\max}$ for the planets of the earth group and Jupiter (in degrees) on the value of the dimensionless asymptotic velocity V_∞ (in km/s)

6 SEARCHING ALGORITHMS FOR THE GRAVITY ASSISTS CHAINS THAT INCREASE THE SC'S ORBIT INCLINATION

The chain GA in the case of their solo execution can be represented as the group of automorphisms V_∞ - sphere. Climbing on the V_∞ - sphere is required to the target point close to the pole of its inclination $\mathrm{T}_{\mathrm{Pole}}$. Arrival to the $\mathrm{T}_{\mathrm{Pole}}$. will mean reaching the geometrically maximum possible inclination of the SC's orbit (1).

According to [1], for missions with $v_\infty = 1/2$ the pole $\mathrm{T}_{\mathrm{Pole}}$ will be localized at the intersection of latitude $\rho = 60^\circ$ and the longitudinal plane σ=0 (Fig. 4).

When conducting an increasing chain of "solo" GA near a fixed planet the V_∞ - sphere is an invariant. Therefore, if necessary, it is possible to solve the problem of constructing a connected route from the starting point of the first GA Λ_0 (contained in the spherical cap $S_{c\varphi,1}$) up to the point of the inclination pole $\mathrm{T}_{\mathrm{Pole}}$ with a sufficiently large number of connecting (in the limiting case, touching) local spherical caps $S_{c\varphi,1}, S_{c\varphi,2}, ..., S_{c\varphi,N}$ [2], (Fig. 4, green circles).

The maps must overlap. Each $S_{c\varphi,2},, S_{c\varphi,N-1}$ must contain at least the two points of different resonance lines between the orbital periods of planets and SC (GA output), which provides the construction of a new SC to the planet after a short time.

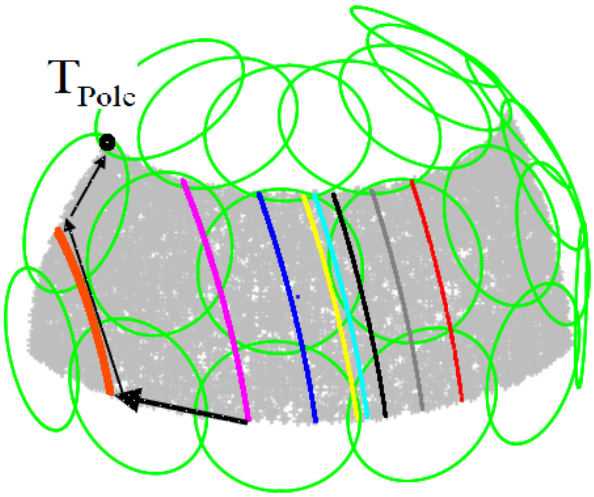

Fig. 4. For missions of any class $i_{\max} = i_{required}$, the inclination pole will be at latitude $\rho = \pi - i_{required}$. Isolines of the resonant ratios between the SC's orbital periods and the planet of the following types are plotted: 1:2, 3:4, 1:1 (blue line), 5:4, 4:3, 3:2, 2:1, 3:1

7 CONCLUSIONS

The formulas obtained in [1-3] for the orbital inclination of the spacecraft and its change as a result of gravity assist maneuvers (GM) around the planet are generalized for the case when not only the spacecraft's orbit, but also the planet's orbit is elliptical. The geometric interpretation of the formulas is described. Formulas are obtained for the coordinates of the inclination pole on the invariant sphere of the asymptotic velocity of the spacecraft. A comparative analysis of the results and modern descriptions of spatial 3D GMs was carried out [4-8]. Some cases are described when A. Labunsky's approximation [4] is acceptable [1-3].

A comparative analysis of the results and modern descriptions of spatial 3D GM is carried out [4-8]. The procedure for constructing a GM chains leading to the inclination pole to achieve a geometrically acceptable maximum inclination of the spacecraft's orbit is analytically investigated.

The characteristic "working" size of the spherical region of the elementary GM on the surface of the sphere is determined. An algorithm for searching for ballistic scenarios is presented, which reduces to constructing a finite simply connected chain GM from the initial GM to the inclination pole, covered with spherical caps on the resonant lines of the invariant sphere. These chains can go either along the resonant isolines, or jumping between them. As a result, a formalized structure of GMS that increase the inclination of the SC's orbit is synthesized, which allows automating the process of adaptive synthesis of phase beams of the corresponding optimal trajectories consisting of millions of variants.

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