

ON THE MATHEMATICAL MODEL OF COMBINED RAREFACTION AND COMPRESSION WAVES IN CONDENSED MATTER

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Summary. Two waves model where shock wave is combined with rarefaction wave appearing in laser ablation due to metal-nonmetal transition effect is investigated using conservation laws for mass and momentum fluxes for the steady-state regime of the process. This approach permits to obtain the relation between front velocities of the waves which shows that the rarefaction wave can be rather slow compared with the generated shock wave.

1. INTRODUCTION

Compression shock waves with supersonic propagation speed are well known in mathematics and physics due to its practical importance and rather simple generation [1-3]. Rarefaction shock waves are not so widespread phenomena because, in particular, for its generation more special conditions are needed [3-6]. Laser ablation due to surface vaporization process can be considered as an example of slow speed rarefaction waves moving into the irradiated condensed matter. In [7] it was suggested that during laser metal ablation “induced transparency wave” arisen from metal-nonmetal transition (MNT) [8] can propagate into some metals which, in contrast to vaporization process, remain in liquid state with diminished density. The laser ablation with possible MNT effect is considered in many papers (see, e.g. [9-11] and references therein) without sufficient attention to hydrodynamic aspects of the problem.

In the present paper some properties of combined compression and rarefaction waves are investigated which, to our knowledge, have not been discussed before.

2. STATEMENT OF THE PROBLEM

In the considered condensed matter (liquid) which was initially at rest in the half-space $z \geq 0$ with pressure P_0 , density ρ_0 and velocity $V_0 = 0$ two combined compression and rarefaction waves are propagating with constant velocities, respectively, $D > d > 0$. The rarefaction wave movement is due to MNT effect mentioned above. Between compression and rarefaction wave fronts one has for velocity, pressure, and density the relations: $V_1 > 0$, $P_1 > P_0$, and $\rho_1 > \rho_0$ while after rarefaction wave front $V_2 < 0$, $P_2 > P_0$, and $\rho_c < \rho \leq \rho_2$, where ρ_c means critical density for liquid-vapor phase transition and P_2 at the irradiated surface depends on the metal ablation regime conditions.

Conservation laws for mass and momentum fluxes at the two fronts are as follows:

$$\rho D = \rho_1(D - V_1) \quad (1)$$

$$P + \rho D^2 = P_1 + \rho_1(D - V_1)^2 \quad (2)$$

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$$\rho_1(d - V_1) = \rho_2(d - V_2) \quad (3)$$

$$P_1 + \rho_1(d - V_1)^2 = P_2 + \rho_2(d - V_2)^2 \quad (4)$$

From these equation it is possible to obtain useful relation between velocities of shock and rarefaction waves.

3. RESULTS AND DISCUSSION

To obtain the relation between d and D it is necessary to exclude from (1)-(4) velocities V_1 , V_2 and pressure P_1 , which can be done in a straightforward manner. From (1), (2) it follows:

$$D \cdot V_1 = D \cdot B_{01} \quad (5)$$

$$P = P_1 - \rho D^2(1 - B_{01}) \quad (6)$$

where $B_{01} = \rho_0/\rho_1$. Taking into account eqs. (1), (2) and notation $B_{12} = \rho_1/\rho_2$ one obtains for P_1 :

$$P_1 = P_2 + \rho_1(d - V_1)^2(B_{12} - 1) = P_2 + \rho_1[d - D(1 - B_{01})]^2(B_{12} - 1) \quad (7)$$

From (6), (7) then it follows:

$$P = P_2 + \rho_1(B_{12} - 1)[d - D(1 - B_{01})]^2 - \rho D^2(1 - B_{01}) \quad (8)$$

After using the relation $m = d/D$ one can rewrite (8) in the form:

$$\left\{ \frac{\left[\frac{P_2 - P_0}{\rho_1 D^2} - B_{01}(1 - B_{01}) \right]}{1 - B_{12}} \right\} = [(1 - B_{01}) - m]^2 \quad (9)$$

$$\left\{ \frac{\left[\frac{P_2 - P_0}{\rho_0 D^2} - (1 - B_{01}) \right] B_{01}}{1 - B_{12}} \right\} = [(1 - B_{01}) - m]^2 \quad (9a)$$

It is clear that in (9) the expression in brackets $\{ \}$ cannot be negative while its numerator cannot be positive because $(1 - B_{12}) < 0$. At the threshold where the expression in brackets is zero one has for maximum value of B_{01M} and corresponding value of m_M :

$$(1 - B_{01M}) = \frac{P_2 - P_0}{\rho_0 D^2} \quad (10)$$

$$m_M = (1 - B_{01M}) \quad (11)$$

These equations permit to estimate approximately the threshold value of m_M and d for the experiment conditions [9,10] because in this case D differs from sound velocity but slightly. For example, at $P_2 - P_0 \approx 300$ bar ($3 \cdot 10^8$ g/s²cm) and $\rho_0 D^2 \approx 3 \cdot 10^{11}$ g/s²cm this gives $m_M \approx 10^{-3}$. Above the threshold evolution of m is determined by the expression:

$$m = 1 - B_{01} \pm \left\{ \frac{B_{01} \left[\frac{(P_2 - P_0)}{\rho_0 D^2} + B_{01} - 1 \right]}{1 - B_{12}} \right\}^{\frac{1}{2}} \quad (12)$$

where the solution with sign (+) is appropriate for $B_{01} < B_{01M}$ due to the condition $m > 0$ while it is not so for the second solution with sign (-). Dependence of both solutions (12)

on B_{01} is shown in Fig.1 at different values of B_{12} and constant other parameters for simplicity. It should be mentioned that B_{01} , B_{12} , D and P_2 vary in accordance with equation of state and with MNT properties as well as with laser ablation regime which determines also value of d .

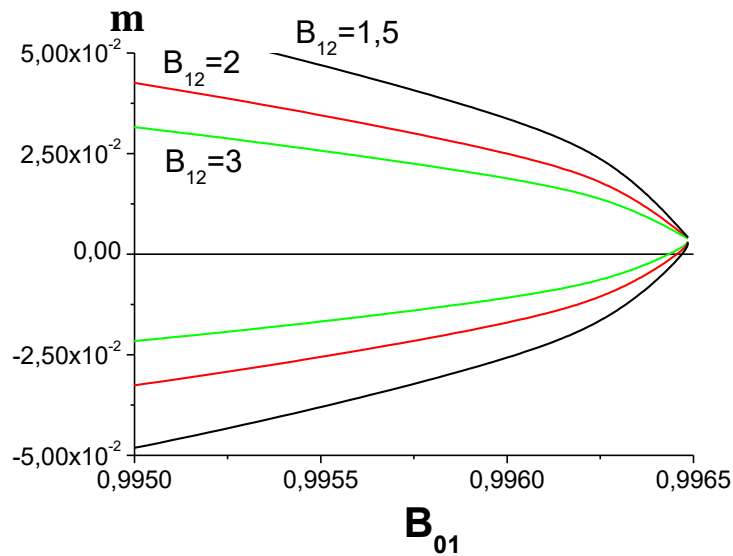


Fig.1 Dependencies of the value m on the ratio B_{01} for three fixed ratio B_{12} and one fixed ratio $(P_2 - P_0)/\rho_0 D^2 = 1 - B_{01M} = 3.5 \cdot 10^{-3}$

4. CONCLUSION

Presented here investigation of mathematical properties pertinent to suggested model of combined compression and rarefaction waves permits to obtain the relation between the propagation velocities of the waves. The investigation is based on analysis of conservation laws for the steady-state mass and momentum fluxes. More detailed information on the considered regime can be obtained taking into account the energy flux conservation law as well as using time-dependent mathematical modeling of laser ablation with MNT effect.

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