

**A SHORT SURVEY OF SOME TOPOLOGIES
ON PRIVALOV SPACES ON THE UNIT DISK**

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*Dedicated to the memory of
Prof. Valerian Ivanovich Gavrilov (1935 – 2016)*

Summary. For $1 < p < \infty$, the Privalov class N^p consists of all holomorphic functions f on the open unit disk \mathbb{D} of the complex plane \mathbb{C} such that

$$\sup_{0 \leq r < 1} \int_0^{2\pi} (\log^+ |f(re^{i\theta})|)^p \frac{d\theta}{2\pi} < +\infty.$$

It was proved by M. Stoll [37] that the space N^p with the topology given by the metric ρ_p defined as

$$\rho_p(f, g) = \left(\int_0^{2\pi} (\log(1 + |f^*(e^{i\theta}) - g^*(e^{i\theta})|))^p \frac{d\theta}{2\pi} \right)^{1/p}, \quad f, g \in N^p,$$

becomes an F -algebra.

In this overview paper we provides a survey of some known results on the topological structures of the Privalov classes N^p ($1 < p < \infty$) and their Fréchet envelopes F^p ($1 < p < \infty$). In particular, we present some results that compare different topologies on the classes N^p ($1 < p < \infty$).

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1 INTRODUCTION

In this overview paper we give a survey of some known results on different topologies of the Privalov classes N^p ($1 < p < \infty$) and their Fréchet envelopes F^p ($1 < p < \infty$) on the open unit disk. Let \mathbb{D} denote the open unit disk in the complex plane and let \mathbb{T} denote the boundary of \mathbb{D} . Let $L^q(\mathbb{T})$ ($0 < q \leq \infty$) be the familiar *Lebesgue spaces* on \mathbb{T} . The *Nevanlinna class* N consists of all holomorphic functions f on \mathbb{D} such that

$$\sup_{0 \leq r < 1} \int_0^{2\pi} \log^+ |f(re^{i\theta})| \frac{d\theta}{2\pi} < \infty,$$

where $\log^+ |x| = \max(\log |x|, 0)$ for $x \neq 0$ and $\log^+ 0 = 0$.

It is well known that for each $f \in N$, the *radial limit* (the *boundary value*) of f defined as

$$f^*(e^{i\theta}) = \lim_{r \rightarrow 1} f(re^{i\theta})$$

exists for almost every $e^{i\theta} \in \mathbb{T}$ (see, e.g., [8, p. 97]).

The *Smirnov class* N^+ consists of those functions $f \in N$ for which

$$\lim_{r \rightarrow 1} \int_0^{2\pi} \log^+ |f(re^{i\theta})| \frac{d\theta}{2\pi} = \int_0^{2\pi} \log^+ |f^*(e^{i\theta})| \frac{d\theta}{2\pi} < \infty.$$

Let us recall that we denote by H^q ($0 < q < \infty$) the classical *Hardy space* on \mathbb{D} consisting of all holomorphic functions f on \mathbb{D} such that

$$\|f\|_q^{\max\{1, q\}} := \sup_{0 \leq r < 1} \int_0^{2\pi} |f(re^{i\theta})|^q \frac{d\theta}{2\pi} < +\infty.$$

Further, H^∞ is the *space of all bounded holomorphic functions* on \mathbb{D} with the supremum norm $\|\cdot\|_\infty$ defined as

$$\|f\|_\infty = \sup_{z \in \mathbb{D}} |f(z)|, \quad f \in H^\infty.$$

We refer [4] for a good reference on the spaces N^+ and H^q ($0 < q \leq \infty$).

The *Privalov class* N^p ($1 < p < \infty$) consists of all holomorphic functions f on \mathbb{D} for which

$$\sup_{0 \leq r < 1} \int_0^{2\pi} (\log^+ |f(re^{i\theta})|)^p \frac{d\theta}{2\pi} < +\infty.$$

These classes were introduced in the first edition of Privalov's book [33, p. 93], where N^p is denoted as A_p . It is known [32] (also see [22, Section 3]) that

$$N^q \subset N^p \quad (q > p), \quad \bigcup_{p>0} H^p \subset \bigcap_{p>1} N^p, \quad \text{and} \quad \bigcup_{p>1} N^p \subset N^+,$$

where the above inclusion relations are proper.

The study of the spaces N^p ($1 < p < \infty$) was continued in 1977 by M. Stoll [37] (with the notation $(\log^+ H)^\alpha$ in [37]). Further, the linear topological and functional properties

of these spaces were extensively investigated by C.M. Eoff in [5] and [6], N. Mochizuki [32], Y. Iida and N. Mochizuki [11], Y. Matsugu [13], J.S. Choa [2], J.S. Choa and H.O. Kim [3], A.K. Sharma and S.-I. Ueki [35] and in works [16]–[31] of authors of this paper; typically, the notation varied and Privalov was mentioned in [13], [26], [28] [30], [31] and [35]. In particular, it was proved in [18, Corollary] that N^p is not locally convex space and in [27, Theorem 1.1] that N^p is not locally bounded space. Furthermore, linear topological and algebraic properties of the spaces N^p and their Fréchet envelopes were recently investigated in [19], [20], [26] and [31]. We refer the recent monograph [9, Chapters 2, 3 and 9] by V.I. Gavrilov, A.V. Subbotin and D.A. Efimov for a good reference on the spaces N^p ($1 < p < \infty$).

The remainder of this overview paper is organized in three sections. For any fixed $p > 1$, in Section 2 we give reformulations of some known results concerning comparisons of different topologies defined on Privalov class N^p and its Fréchet envelope F^p . Section 3 is devoted to the considerations of Helson topology and Szegő topology on the space N^p and comparisons of these topologies with topologies given in Section 2. Concluding remarks are given in the last section of the paper.

2 THE STOLL'S METRIC TOPOLOGY ON N^p AND THE FRÉCHET ENVELOPE OF THE SPACE N^p

In 1977 M. Stoll [37] proved the following basic result.

Theorem 1 ([37, Theorem 4.2]). *The Privalov space N^p ($1 < p < \infty$) (with the notation $(\log^+ H)^p$ in [37]) with the topology given by the metric ρ_p defined as*

$$\rho_p(f, g) = \left(\int_0^{2\pi} (\log(1 + |f(e^{i\theta}) - g(e^{i\theta})|))^p \frac{d\theta}{2\pi} \right)^{1/p}, \quad f, g \in N^p, \quad (1)$$

is an F -algebra, i.e., an F -space (a complete metrizable topological vector space with the translation invariant metric) which is an algebra in which multiplication is continuous.

Remark 1. Notice that (1) with $p = 1$ defines the metric ρ_1 on the Smirnov class N^+ introduced in 1973 by N. Yanagihara [39]. It was proved in [39] that the metric ρ_1 induces the topology on N^+ under which N^+ is also an F -algebra.

The following result immediately follows from the inequality (4.3) in [37].

Theorem 2 (see [17, Chapter 2, Lemma 2.1]). *The convergence in the metric ρ_p of the space N^p defined by (1) is stronger than the metric of uniform convergence on compact subsets of the disk \mathbb{D} .*

In connection with the space N^p ($1 < p < \infty$), Stoll in [37] also studied the space F^q ($0 < q < \infty$) with the notation $F_{1/q}$ in [37], consisting of those functions f holomorphic on \mathbb{D} for which

$$\lim_{r \rightarrow 1} (1 - r)^{1/q} \log^+ M_\infty(r, f) = 0,$$

where

$$M_\infty(r, f) = \max_{|z| \leq r} |f(z)|, \quad 0 \leq r < 1.$$

Here, as always in the sequel, we will assume that $q = p$ is any fixed real number greater than 1.

Stoll [37] proved the following result.

Theorem 3 ([37, Theorem 2.2]). *Suppose that $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is a holomorphic function on \mathbb{D} . Then the following statements are equivalent:*

- (a) $f \in F^p$;
- (b) *there exists a sequence $\{c_n\}_{n=0}^{\infty}$ of positive real numbers with $c_n \rightarrow 0$ satisfying the condition*

$$|a_n| \leq \exp(c_n n^{1/(p+1)}), \quad n = 0, 1, 2, \dots;$$

- (c) *for any $c > 0$,*

$$\|f\|_{p,c} := \sum_{n=0}^{\infty} |a_n| \exp(-cn^{1/(p+1)}) < \infty. \quad (2)$$

Notice that in view of Theorem 3 ((a) \Leftrightarrow (c)), by (2) it is well defined the family of seminorms $\{\|\cdot\|_{p,c}\}_{c>0}$ on F^p .

Moreover, Stoll [37] also considered the family of seminorms $\{\|\|\cdot\|\|_{p,c}\}_{c>0}$ on F^p given as

$$\|\|f\|\|_{p,c} = \int_0^1 \exp(-c(1-r)^{-1/p}) M_p(r, f) dr, \quad f \in F^p, \quad (3)$$

where

$$M_p(r, f) = \left(\int_0^{2\pi} |f(re^{i\theta})|^p \frac{d\theta}{2\pi} \right)^{1/p}.$$

By [37, Proposition 3.1], $\{\|\cdot\|_{p,c}\}_{c>0}$ and $\{\|\|\cdot\|\|_{p,c}\}_{c>0}$ are equivalent families of seminorms, and hence, they induce the same topology on the space F^p . More precisely, the following estimates have been proved in [37].

Theorem 4 ([37, Proposition 3.1]). *For each $c > 0$, there is a constant A depending only on p and c , such that*

$$\|\|f\|\|_{p,c} \leq \|f\|_{p,c_1} \quad \text{and} \quad \|f\|_{p,c} \leq A \|\|f\|\|_{p,c_2},$$

with $c_1 = c^{p/(p+1)}$ and $c_2 = (\frac{c}{12})^{p/(p+1)}$.

Let us recall that a locally convex F -space is called a *Fréchet space*, and a *Fréchet algebra* is a Fréchet space that is an algebra in which multiplication is continuous.

It was proved in [37, Theorem 3.2] that the space F^p with the topology induced by the family of seminorms $\{\|\cdot\|_{p,c}\}_{c>0}$ or $\{\|\|\cdot\|\|_{p,c}\}_{c>0}$, is a countably normed Fréchet algebra. Moreover, there holds

$$\|fg\|_{p,c} \leq \|f\|_{p,c'} \|g\|_{p,c'}, \quad \text{for all } f, g \in F^p,$$

where $c' = c \cdot 2^{-p/(p+1)}$.

The most important connection between the spaces N^p and F^p is given by the following result.

Theorem 5 ([37, Theorem 4.3]). *For any fixed $p > 1$ the following assertions hold:*

- (a) N^p is a dense subspace of F^p in the topology defined by the family of seminorms (2) or (3);
- (b) the topology on F^p defined by the family of seminorms (2) or (3) is weaker than the topology on N^p induced by the metric ρ_p defined by (1);
- (c) for each $q > p$ there exists a function $f_q \in N^p$ such that

$$\limsup_{r \rightarrow 1} (1-r)^{1/q} \log^+ M_\infty(r, f_q) > 0,$$

i.e., N^p is not contained in F^q for none $q > p$.

Remark 2. Recall that the spaces F^p have also been studied independently by A.I. Zayed in [41] and [42]; many of the results in [40] parallel those of Stoll in [37], albeit in a more general setting. For $p = 1$, the space F_1 has been denoted by F^+ and has been studied by Yanagihara in [39] and [40] and Stoll in [36]. It was shown in [39] and [40] that F^+ is actually the containing Fréchet space for N^+ , i.e., N^+ with the initial topology embeds densely into F^+ , under the natural inclusion, and F^+ and N^+ have the same topological dual.

Observe that the space F^p topologized by the family of seminorms $\{\|\cdot\|_{p,c}\}_{c>0}$ given by (2) is metrizable by the metric λ_p defined as

$$\lambda_p(f, g) = \sum_{n=1}^{\infty} 2^{-n} \frac{\|f - g\|_{p,1/n^{p/(p+1)}}}{1 + \|f - g\|_{p,1/n^{p/(p+1)}}}, \quad f, g \in F^p. \quad (4)$$

Namely, it is easy to show that the above metric λ_p and the family of seminorms $\{\|\cdot\|_{p,c}\}_{c>0}$ induce the same topology on the space F^p .

Let us recall that if $X = (X, \tau)$ is an F -space whose topological dual (the set of all continuous linear functionals on X) X^* separates the points of X , then its *Fréchet envelope* \widehat{X} is defined to be the completion of the space (X, τ^c) , where τ^c is the strongest locally convex (necessarily metrizable) topology on X that is weaker than τ . In fact, it is known that τ^c is equal to the *Mackey topology* of the dual pair (X, X^*) , i.e., to the unique maximal locally convex topology on X for which X still has dual space X^* (see [34, Theorem 1]). For each metrizable locally convex topology τ on X , (X, τ) is a *Mackey space*, i.e., τ coincides with the Mackey topology of the dual pair (X, X^*) (see [12, Corollary 22.3, p. 210]).

Eoff [5, proof of Theorem 4.2] proved that the topology of F^p with $1 < p < \infty$, (respectively, $F^1 := F^+$) is stronger than that of the Fréchet envelope of N^p (respectively, N^+). As an immediate consequence of this result, it was obtained the following important assertion.

Theorem 6 ([5, Theorem 4.2, the case $p > 1$]). *For each $p > 1$, F^p is the Fréchet envelope of N^p .*

3 HELSON TOPOLOGY AND SZEGŐ TOPOLOGY ON THE SPACE N^p

It is well known [4, p. 26] that a function f holomorphic on \mathbb{D} belongs to the Smirnov class N^+ if and only if $f = IF$, where I is an inner function on \mathbb{D} and F is an outer function given by

$$F(z) = \exp \left(\int_0^{2\pi} \frac{e^{it} + z}{e^{it} - z} \log |F^*(e^{it})| \frac{dt}{2\pi} \right), \quad z \in \mathbb{D},$$

where $\log |F^*| \in L^1(T)$.

Privalov [33, p. 98] showed that a function f holomorphic on \mathbb{D} belongs to the class N^p if and only if $f = IF$, where I is an inner function on \mathbb{D} and F is an outer function as given above with $\log^+ |f^*| \in L^p(T)$ (or equivalently, $\log^+ |F^*| \in L^p(T)$). It follows immediately from this factorization that the set $(N^p)^{-1}$ of all invertible elements in N^p consists of those outer functions F for which $\log |F^*| \in L^p(T)$. This result was used in [6] to prove that a function f belongs to the class N^p if and only if it can be expressed as the ratio g/h , where g and h are in H^2 and h belongs to the set $(N^p)^{-1}$. Using this fact and the famous Beurling's theorem for the Hardy space H^2 ([1]; also see [10, Ch. 7, p. 99]), it is easy to show that (see [6])

$$N^p = \bigcup_{h \in (N^p)^{-1}} H^2(|h^*|^2), \quad (5)$$

where $H^2(|h^*|^2)$ denotes the closure of analytic polynomials in Lebesgue space $L^2(|h^*|^2)$. The corresponding result for the class N^+ was obtained in [15, p. 230]; also see [7, V.4.4], where $(N^+)^{-1}$ is the set of all outer functions.

The representation (5) allows one to define two topologies on N^p [24]: the usual locally convex inductive limit topology, which we shall call the *Helson topology* and denote \mathcal{H}_p , in which a neighborhood base for 0 is given by those balanced convex sets whose intersection with each $H^2(|h^*|^2)$ is a neighborhood of zero in $H^2(|h^*|^2)$, and a not locally convex topology, which we shall denote I_p , in which a neighborhood base for zero is given by all sets whose intersection with each space $H^2(|h^*|^2)$ is a neighborhood of zero. It was proved in [6] the following result.

Theorem 7 ([6]). *For each $p > 1$ the topology I_p on the space N^p coincides with the metric topology ρ_p defined on N^p by the initial metric given by (1).*

Notice that the analogous result to those of Theorem 7 concerning the Smirnov class N^+ was proved in [14].

Since the metric topology ρ_p is not locally convex, Theorem 7 immediately yields the following result.

Theorem 8 ([24]). *For each $p > 1$ the topology I_p on the space N^p is strictly stronger than the Helson topology \mathcal{H}_p on N^p .*

In [24] it was also proved the following result.

Theorem 9 ([24, the first part of Theorem E]). *The Helson topology \mathcal{H}_p defined on N^p coincides with the metric topology induced on N^p by the metric λ_p given by (4).*

Let us recall that a locally convex topological vector space is called *barrelled* if every closed, absolutely convex, absorbing set is a neighborhood of zero. It is well known that every Banach space is barrelled, and the inductive limit of barrelled spaces is barrelled; so the topological vector space (N^p, \mathcal{H}_p) is barrelled. The most important fact about barrelled spaces is that every pointwise bounded family of continuous linear functionals is equicontinuous. Furthermore, each barrelled space is a Mackey space (see, e.g., [12, pp. 171–173] or [38, 9–3.4]).

Let Taylor expansion at zero of a function f in N^p be given by

$$f(z) = \sum_{k=0}^{\infty} \hat{f}(k)z^k, \quad z \in \mathbb{D}.$$

Then we shall say that a topology τ on N^p is *Szegő* if the functionals $f \mapsto \hat{f}(k)$ ($f \in N^p$), are continuous with respect to this topology for each nonnegative integer k (equivalently, the forward and backward shifts are continuous). In particular, the Helson topology \mathcal{H}_p and the metric topology ρ_p defined on N^p are Szegő. The following result which was proved in [24] is the N^p -analogue of Theorem 2.1 in [15].

Theorem 10 ([24, Theorem 1]). *Let τ be a barrelled Szegő topology on the space N^p , \mathcal{H}_p the Helson topology on N^p and let λ_p be the metric topology on N^p induced by the metric given by (4). Then the following statements are equivalent:*

- (i) $\tau = \mathcal{H}_p$;
- (ii) $\tau = \lambda_p$;
- (iii) *Fourier series converge in the space (N^p, τ) .*

4 CONCLUSION

A survey of some results on the topological structures of the Privalov classes N^p ($1 < p < \infty$) on the open unit disk and their Fréchet envelopes F^p ($1 < p < \infty$) is provided in this overview paper. Let us recall that presented results on considered topologies on the spaces N^p and F^p are mainly obtained in papers [6], [24] and [37]. All these results are generalizations of results earlier obtained for the Smirnov class N^+ .

Notice also that the assertions in this paper have a basic significance for the study of linear topological and algebraic properties of F -algebras N^p and F^p . Moreover, some of these assertions, as well as their N^+ -analogues, are already being applied for proving some approximation-type theorems in Functional Analysis ([15] and [24]) and some results in Operator Theory ([14] and [25]). Accordingly, we believe that different approaches to the topological structures of the spaces N^p and F^p would be of interest for further investigations in this field, including some applications.

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