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ON PARALLEL ALGORITHMS FOR SOLVING THE DIRECT AND INVERSE PROBLEMS OF IONOSPHERIC SOUNDING

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Summary. The article is devoted to solving the direct and inverse problems of sounding the ionosphere. Parallel numerical methods and fast algorithms for multiprocessor computing system with common control were developed in order to increase the efficiency of solving these problems. This approach allows to solve the problems in chosen classes of ionospheric models in real time and within errors of measurement.

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1 INTRODUCTION

An inverse problem of ionospheric sounding, i.e. reconstruction of the space structure of the electron concentration, belongs to the class of ill-posed problems of mathematical physics. According to paper[1] the solution of the problem is defined uniquely. The research task is to construct a very complex picture that preserves stability with respect to errors of measurement by the given experimental information. It is clear that the more information is contained in the experimental data the more complicated structures it allows to recognize. For processing big volumes of data in real time, it is required to increase the efficiency of solving the posed problems. One of the directions for solving this problem is the development of fast algorithms which are based on the technique of parallel computations. For arbitrary structure of the ionosphere, the inverse problem can be solved by numerical search methods based on multiple solutions of the direct problem and the application of regularization[2].

Here we will mention two methods of solving the direct problem

1. By using piecewise-analytic solutions for the trajectory parameters of the signal[3].
2. By using numerical methods for solving a system of ordinary differential equations.

We choose the second method in order to avoid a restriction on the class of solvable problems.

2 STATEMENT OF PROBLEM

We consider the problem of radiowave propagation in anisotropic-inhomogeneous ionosphere in geometrical optics approximation. This problem is described by the system of trajectory equations

$$\frac{d\bar{x}}{dt} = M\left(\bar{\xi} - \frac{1}{2}\frac{\partial\varepsilon}{\partial\bar{\xi}}\right)$$
$$\frac{d\bar{\xi}}{dt} = \frac{M}{2}\frac{\partial\varepsilon}{\partial\bar{x}} \tag{1}$$

where $\bar{x}$ is the vector of the signal path coordinates, $\bar{\xi}$ is the normalized wave normal vector, ε is the dielectric constant, t is the group path and

$$M = \frac{1}{\varepsilon + \frac{f}{2}\frac{\partial\varepsilon}{\partial f}}$$

The dielectric constant ε with negligible small quantity of trajectory absorption is defined as

$$\varepsilon = 1 - 2\upsilon(1-\upsilon)\left[2(1-\upsilon) - u\sin^2\vartheta \pm \sqrt{u^2\sin^4\theta + 4u(1-\upsilon)^2\cos^2\theta}\right]^{-1}$$

where $\upsilon = f_N^2/f^2$ and $u = f_H^2/f^2$ are non-dimensional parameters, f_N is the plasma frequency, f_H is the gyrofrequency of electrons, f is the wave frequency, $\bar{H}$ is the vector of magnetic field of the Earth, θ is the angle between $\bar{\xi}$ and $\bar{H}$. The form of the system of the equations was obtained by method of characteristics from the eikonal equation[4,5].

The Cauchy problem of ionospheric sounding is a problem of finding the characteristics of the signal trajectory by solving (1) with given initial output point and the elevation angle of a signal.

The adequacy of the solution of (1) was verified by comparing computed signal parameters for an isotropic quasi-parabolic ionospheric model with the correct values of trajectory parameters calculated by analytic expressions of group path and distance of signal propagation for the same model of the ionosphere. For different sets of parameters for the quasi-parabolic ionosphere model, the error of computation of the trajectory characteristics (1) is the tenth part of a percent. The direct boundary value problem of sounding the ionosphere is finding of signal trajectory characteristics (the group path and the elevation angles) on one-jump sounding trace with fixed bounds and given space structure of electron concentration. Usually this problem is solved by the shooting method.

Solution of the inverse problem of ionospheric sounding defines the space distribution of electronic concentration by radiosounding data (vertical and oblique sounding data).

In order to solve the inverse problem of the composite radiosounding of the ionosphere in accordance with vertical and oblique-incidence sounding ionograms, we were using the ionogram of vertical sounding at the end of the oblique-incidence sounding trace to construct the best approximation of the profile of the initial vertical distribution of the electron concentration. The inverse problem of ionospheric sounding, i.e. the reconstruction of the space structure of electron concentration, belongs to the class of ill-posed problems of mathematical physics. For solving this problem we use the Tikhonov regularization method[1,2].

The space distribution of electron concentration is given as the class of parametric models of the ionosphere. For solving the inverse problem of sounding the ionosphere by the Tikhonov regularization method[1] we introduce a smoothing functional

$$Q_\gamma = Q_\delta + \gamma\Omega$$

which should be minimized. Here

$$Q_\delta = \sum_{l=1}^{L} A[(S_e^l - S_n^l)^2 + (\alpha_e^l - \alpha_n^l)^2 + (\beta_e^l - \beta_n^l)^2]$$

is the residual functional, and

$$\Omega = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(k_i^j\right)^2$$

is the stabilizing functional. The numerical parameter γ is called the regularization parameter. It regulates relations between the functionals Q_δ and Ω. The coefficient A in Q_δ is the normalizing factor, $A = 1/B^2$ where B is the path length. $S_e^l = S(f^l, N_e(z,x))$, $l = 1, \cdots, L$ are the experimental values of the group path for fixed sounding frequencies f^l and the given quasi-experimental two-dimensional model of the electron concentration $N_e(z,x)$; $S_n^l = S(f^l, N_n(z,x))$, $l = 1, \cdots, L$, are the values sought of the group path for the same sounding frequencies and the n-th iteration of the search two-dimensional electron concentration model; α_e^l and β_e^l are the angles of departure and arrival, respectively, and satisfy the boundary conditions for the frequency f^l with the given quasi-experimental model $N_e(z,x)$; α_n^l and β_n^l are the angles of departure and arrival of the ray for the same frequency and the n-th iteration of the search model, which satisfy the boundary conditions; z is the height; x is the distance along the path; L is the number of trajectory rays used.

The vector of the medium parameters

$$\bar{x} = (x_1, \cdots, x_i, \cdots, x_n; k_1^1, \cdots, k_i^1, \cdots, k_i^m, \cdots, k_n^1, \cdots, k_n^m)$$

consists of variables defining the sectors x_i along the trace and the variables k_i^j defining slope of the isolines bounding the sectors $N_n^{ij}(z,x)$ from above and below. Uniform convergence or divergence of the isolines is provided inside each section by linear interpolation of respective coefficients. The isolines have no left and right discontinuities at the boundaries of the sectors $N_n^{ij}(z,x)$. This ensures continuity of $N_n(z,x)$. Uniqueness of $N_n(z,x)$is provided by restrictions on k_i^j that exclude intersections of the bounding isolines inside the sections $N_n^{ij}(z,x)$ during the solution search. Given approach for construction of the function $N_n^{ij}(z,x)$ allows to approximate any inhomogeneous structure of the ionosphere corresponding to the selection of sectors $N_n^{ij}(z,x)$. In our case the functional Ω allows to choose the solution from the set of exiting equivalent solutions that changes most slowly along the path, in other words, with minimum values of k_i^j.

3 FAST ALGORITHMS

We consider two ways of development of fast algorithms for solving posed problems using multiprocessor systems.

In the first approach the algorithm is realized on multiprocessor computing systems as following: input of each processing element is given as a vector of input parameters and is common for all processing elements, the chain of commands is realized by sequential algorithm of posed inverse problem. Since the algorithm of solving the inverse problem is based on multiple time of solving of the direct problem given by system (1) the increase of the number of the processor elements cannot increase the speed of solving the inverse problem.

The second approach takes into account the specific character of considering problems, namely, the form of the system of equations, numerical methods by which these problems are solved. It is found that the sequential algorithm of solving of the given problem has “vector” parts. This property yields to intrinsic parallelization using homogeneous multiprocessor system.

We compared both algorithms and it was obtained that the second parallel algorithm for given class of problems is faster.

Here we describe fast algorithms for the direct Cauchy problem, direct boundary problem and inverse problem according to the second approach.

We rewrite system (1) in a vector form

$$\frac{d\bar{y}}{dt} = M \cdot F(\bar{y})$$

where

$$\bar{y} = (x_1, x_2, x_3, \xi_1, \xi_2, \xi_3)$$

$$F(\bar{y}) = \left(\bar{a} - \frac{\bar{c}}{2} \cdot \frac{\partial \varepsilon}{\partial \bar{y}}\right)$$

$$\bar{a} = (\xi_1, \xi_2, \xi_3, 0,0,0)$$

$$\bar{c} = (-1, -1, -1_3, 1,1,1)$$

Parallel algorithms of solving the direct Cauchy problem are based on simultaneous solving of all equations of the vector system. In this case it is enough to use only one segment of a computing field of a homogeneous multiprocessor (eight processing elements (PE): six PEs are used for solving the system, two PEs - for computing the system coefficients).

Segment of the computing system

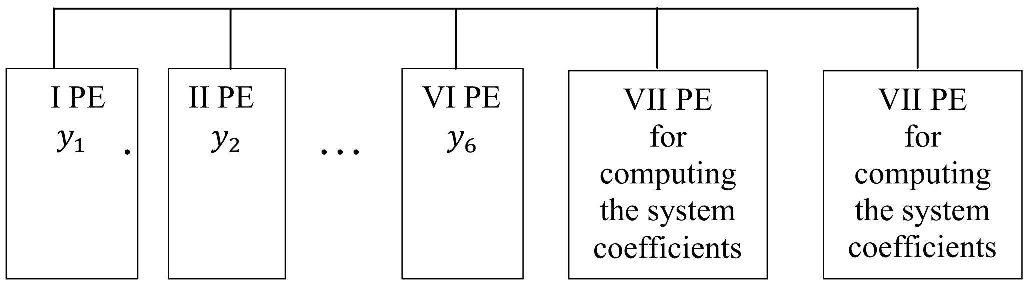

Fig.1. Parallel algorithm for solving the direct Cauchy problem

The parallel algorithm for solving the direct Cauchy problem parallelizes independent procedures of the sequential algorithm: procedure of numerical method for solving the system of trajectory equations (1), procedure of finding the right-hand sides of the system, procedure for finding the index of reflection ε. The possibility of application of the standard parallel numerical method of solving of ordinary differential equations systems in more general cases was studied and the preference of the constructed parallel numerical method was obtained.

For solving the boundary problem we consider a parallel numerical shooting method (similar to the standard shooting method). For a fixed signal frequency we set the series of m different angles of elevation and realize the parallel algorithm of the direct Cauchy problem on each of m segments of multiprocessor computing field. If $m \geq 4$ then it is found that usually one step of parallel shooting method is enough to find the angle of elevation which satisfies the given signal frequency.

For efficiency of solving the inverse problem of sounding the ionosphere we developed a parallel algorithm oriented on a particular architecture of a multiprocessor computing system consisting of several homogeneous multiprocessors. The parallel numerical method of minimization of the residual functional was realized on all multiprocessors of the system; the parallel numerical method of solving the direct boundary problem was realized on each multiprocessor; and the parallel algorithm of solving the direct Cauchy problem was realized on each segment of computing field on all multiprocessors.

Remark. A computing system consists of n multiprocessors, each multiprocessor consists of m segments, and each segment consists of 8 processing elements (PE).

Our algorithm was realized on a computing system which in general can be described as following:

Computing field of multiprocessor

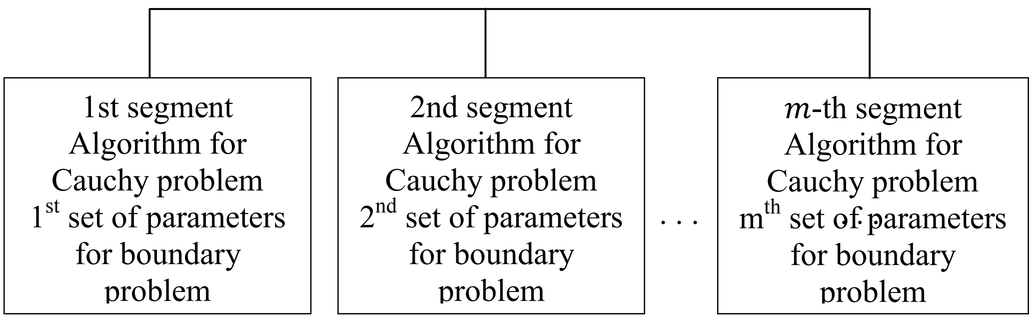

Fig.2. Parallel algorithm for solving the direct boundary problem

Computing system

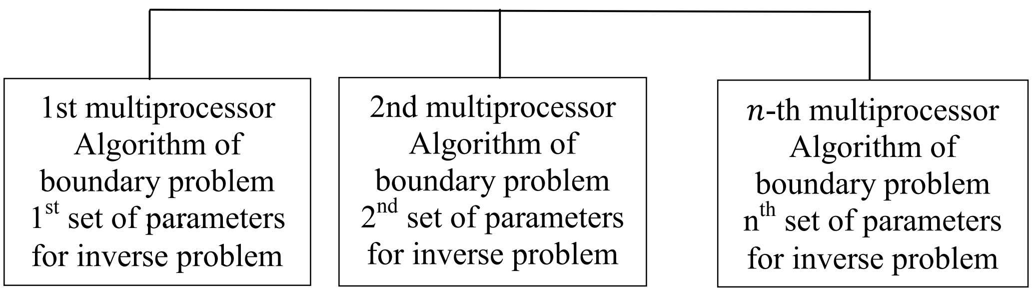

Fig.3. Parallel algorithm for solving the inverse problem

4 EXAMPLE OF SOLVING THE INVERSE PROBLEM

The fast algorithms of solving the direct and inverse problems of ionospheric sounding were realized in the case of isotropic approximation for reconstruction of two-dimensional inhomogeneous structure of the ionosphere on homogeneous multiprocessing system with computing field consisting of one segment.

The developed algorithms were tested for reconstruction of quasi-experimental heterogeneities which were synthesized for the specific realizations of the two-dimensional section of the International Ionosphere Model provided by IRI.

The class of parametric models of the two-dimensional ionosphere was defined as a class of piecewise linear approximations (in the spherical coordinates) of the electron concentration function providing continuity and uniqueness of the plasma frequency function in the sounding area. The algorithm for solving the inverse problem uses the vertical distribution of electron concentration on both ends of the sounding trace and is synthesized with characteristics of oblique sounding - group delay and angles of elevation on the trace ends. The class of multi-parametric ionospheric models was constructed as the class of functions with piecewise-linear variation of values of the electron concentration function on each interval of partition of the ionosphere model along the trace. The isolines of the electron concentration of approximating ionosphere were almost the same as isolines of electron concentration of the quasi-experimental model.

Using the data of the complex experiment of vertical and backscatter oblique sounding we obtained the estimate of two-dimensional structure. The algorithm of solving the inverse problem uses as initial data the ionograms of vertical sounding on both ends of the trace and the frequency characteristics of group delay of oblique sounding. The ionograms of vertical sounding on the trace ends were used for the best approximation of the vertical profile of electron concentration distribution which then becomes fixed. Three types of experimental data were analyzed for reconstruction of real structure of the ionosphere in accordance with composite experimental data of vertical and oblique sounding: homogeneous, weak-inhomogeneous and strong-inhomogeneous ionosphere structure along the sounding trace.

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