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Dedicated to our Professor and friend Dr. V. I. Gavrilov
on the occasion of his 80th birthday

A SHORT SURVEY OF THE IDEAL STRUCTURE OF PRIVALOV SPACES ON THE UNIT DISK

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Summary. For $1 < p < \infty$, the Privalov class N^p consists of all holomorphic functions f on the open unit disk $\mathbb{D}$ of the complex plane $\mathbb{C}$ such that

$$\sup_{0 \le r < 1} \int_0^{2\pi} (\log^+ |f(re^{i\theta})|)^p \, \frac{d\theta}{2\pi} < +\infty.$$

M. Stoll [32] showed that the space N^p with the topology given by the metric d_p defined as

$$d_p(f,g) = \Big(\int_0^{2\pi} \big(\log(1 + |f^*(e^{i\theta}) - g^*(e^{i\theta})|) \big)^p \, \frac{d\theta}{2\pi} \Big)^{1/p}, \quad f, g \in N^p,$$

becomes an F-algebra.

In this overview paper we give a survey of some known results related to the ideal structure of Privalov classes N^p $(1 < p < \infty)$. In Section 2 we point out that every space N^p $(1 < p < \infty)$ is a ring of Nevanlinna–Smirnov type in the sense of Mortini [27]. Consequently, in the next section we establish the facts that N^p is a coherent ring and that N^p has the Corona Property. In Section 4 we present a result of N. Mochizuki [26] which gives a complete characterization of the closed ideals in N^p. Consequently, if $\mathcal{M}$ is a closed ideal in N^p which is not identically 0, then there is a unique modulo constants

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inner function φ such that $\mathcal{M} = \varphi N^p$. Using this result, it can be proved that a closed subspace E of N^p is invariant if and only if it has the form φN^p for some inner function φ. This result is in fact the N^p-analogue of the famous Beurling's theorem for the Hardy spaces H^q $(0 < q < \infty)$.

1 INTRODUCTION

Let $\mathbb{D}$ denote the open unit disk in the complex plane and let $\mathbb{T}$ denote the boundary of $\mathbb{D}$. Let $L^q(\mathbb{T})$ $(0 < q \leqslant \infty)$ be the familiar Lebesgue spaces on $\mathbb{T}$. The *Nevanlinna class* N is the set of all functions f holomorphic on $\mathbb{D}$ such that

$$\sup_{0\le r<1}\int_0^{2\pi}\log^+\left|f\left(re^{i\theta}\right)\right|\frac{d\theta}{2\pi}<\infty,$$

where $\log^+|x| = \max(\log|x|, 0)$ for $x \neq 0$ and $\log^+ 0 = 0$.

It is well known that for each $f \in N$, the *radial limit* (the *boundary value*) of f defined as

$$f^*(e^{i\theta}) = \lim_{r\to 1} f(re^{i\theta})$$

exists for almost every $e^{i\theta} \in \mathbb{T}$ (e.g., see [7, p. 97]).

The *Smirnov class* N^+ consists of those functions $f \in N$ for which

$$\lim_{r\to 1}\int_0^{2\pi}\log^+\left|f\left(re^{i\theta}\right)\right|\frac{d\theta}{2\pi}=\int_0^{2\pi}\log^+\left|f^*\left(e^{i\theta}\right)\right|\frac{d\theta}{2\pi}<\infty.$$

Recall that we denote by H^q $(0 < q \le \infty)$ the classical *Hardy space* on $\mathbb{D}$, defined as the set of all holomorphic functions f on $\mathbb{D}$ for which

$$\|f\|_q^{\max\{1,q\}} := \sup_{0\le r<1}\int_0^{2\pi}|f(re^{i\theta})|^q\,\frac{d\theta}{2\pi}<+\infty.$$

Further, H^∞ is the *space of all bounded holomorphic functions* on $\mathbb{D}$ with the supremum norm $\|\cdot\|_\infty$ defined as

$$\|f\|_\infty = \sup_{z\in\mathbb{D}}|f(z)|, \quad f \in H^\infty.$$

We refer [4] for a good reference on the spaces H^q and N^+.

For $(1 < p < \infty)$ the *Privalov class* N^p consists of all holomorphic functions f on $\mathbb{D}$ for which

$$\sup_{0\le r<1}\int_0^{2\pi}(\log^+|f(re^{i\theta})|)^p\,\frac{d\theta}{2\pi}<+\infty.$$

These classes were introduced in the first edition of Privalov's book [28, p. 93], where N^p is denoted as A_p. It is known [26] (also see [19, Section 3]) that

$$N^q \subset N^p \ (q>p), \quad \bigcup_{p>0} H^p \subset \bigcap_{p>1} N^p, \quad \text{and} \quad \bigcup_{p>1} N^p \subset N^+,$$

where the above containment relations are proper.

The study of the spaces N^p $(1 < p < \infty)$ was continued in 1977 by M. Stoll [32] (with the notation $(\log^+ H)^\alpha$ in [32]). Further, the topological and functional properties of these spaces were studied by C.M. Eoff ([5] and [6]), N. Mochizuki [26], Y. Iida and N. Mochizuki [10], Y. Matsugu [12], J.S. Choa [2], J.S. Choa and H.O. Kim [3], A.K. Sharma and S.-I. Ueki [30] and in works [19]–[25] of authors of this paper; typically, the notation of these spaces varied. Linear topological structure of the spaces N^p and their Fréchet envelopes was investigated in [16], [17], [21] and [22]. In particular, it was proved in [16, Theorem] that the space N^p $(1 < p < \infty)$ does not have the *Hahn-Banach approximation property*, and hence, it does not have the *Hahn-Banach separation property*. Furthermore, the spaces N^p are neither *locally convex* [16, Corollary] nor *locally bounded* [23, Theorem 1.1]. Furthermore, the ideal structure of the algebras N^p was investigated in [14], [18], [22] and [26].

We refer the recent monograph [8, Chapters 2, 3 and 9] by V.I. Gavrilov, A.V. Subbotin and D.A. Efimov for a good reference on the spaces N^p.

In 1977 Stoll [32] proved the following result.

Theorem A ([32, Theorem 4.2]). *The Privalov space N^p $(1 < p < \infty)$ (with the notation $(\log^+ H)^p$ in* [32]*) with the topology given by the metric ρ_p defined as*

$$\rho_p(f,g) = \Big(\int_0^{2\pi} \big(\log(1 + |f(e^{i\theta}) - g(e^{i\theta})|) \big)^p \, \frac{d\theta}{2\pi} \Big)^{1/p}, \quad f, g \in N^p, \tag{1}$$

is an F-algebra, i.e., an F-space (a complete metrizable topological vector space with the invariant metric) in which multiplication is continuous.

Notice that (1) with $p = 1$ defines the metric d_1 on the Smirnov class N^+. N. Yanagihara proved [33] that the metric d_1 induces the topology on N^+ under which N^+ is an F-algebra.

It is well known [4, p. 26, Theorem 2.10] that every non-zero function $f \in N^+$ admits a unique factorization of the form

$$f(z) = B(z) S_\mu(z) F(z), \quad z \in \mathbb{D}, \tag{2}$$

where B is the *Blaschke product* with respect to zeros $\{z_n\} \subset \mathbb{D}$ of f (the set $\{z_n\}$ may be finite), S_μ is a *singular inner function*, F is an *outer function* for N^+, i.e.,

$$B(z) = z^m \prod_{n=1}^{\infty} \frac{|z_n|}{z_n} \cdot \frac{z_n - z}{1 - \bar{z}_n z}, \tag{3}$$

with $\sum_{n=1}^{\infty} \big(1 - |z_n|\big) < \infty$, m a nonnegative integer,

$$S_\mu(z) = \exp\left(- \int_0^{2\pi} \frac{e^{it} + z}{e^{it} - z} \, d\mu(t) \right) \tag{4}$$

with a positive singular measure $d\mu$, and

$$F(z) = \lambda \exp \left(\frac{1}{2\pi} \int_0^{2\pi} \frac{e^{it} + z}{e^{it} - z} \log \left| f^*(e^{it}) \right| dt \right), \tag{5}$$

where $|\lambda| = 1$ and

$$\log |f^*(e^{i\theta})| \in L^1(\mathbb{T}). \tag{6}$$

A function F with the factorization (5) and for which $\log |F^*(e^{i\theta})| \in L^1(\mathbb{T})$ is called an *outer function*. Furthermore, a function φ of the form

$$\varphi(z) = B(z) S_\mu(z), \quad z \in \mathbb{D}, \tag{7}$$

where the functions B and S_μ are given by (3) and (4), respectively, is called an *inner function* or the *inner factor* of a function f factorized by (2). Notice that the function φ defined by (7) is a bounded holomorphic function on $\mathbb{D}$ such that $|\varphi^*(e^{i\theta})| = 1$ for almost every $e^{i\theta} \in \mathbb{T}$, and hence, $|f^*(e^{i\theta})| = |F^*(e^{i\theta})|$ for almost every $e^{i\theta} \in \mathbb{T}$.

The inner-outer factorization theorem for the classes N^p is given by Privalov [28] as follows.

Theorem B ([28, pp. 98-100]; also see [6]). *A function $f \in N^+$ factorized by* (2) *with* (3) − (6) *belongs to the Privalov class N^p if and only if* $\log^+ |F^*(e^{i\theta})| \in L^p(\mathbb{T})$.

Remark 1. If we exclude only the condition $\left(\log^+ |F^*|\right)^p \in L^1(T)$ from Theorem B, we obtain the well known canonical factorization theorem for the class N^+ (e.g., see [4, p. 26] or [28, p. 89]).

In this paper, we give a survey of known results related the ideal structure of the Privalov classes N^p $(1 < p < \infty)$.

In Section 2 of [14], the ideal structure of subrings N^p of N with $p > 1$ is described as consequences of the results in [27, Sections 1 and 3] given for an arbitrary ring of Nevanlinna–Smirnov type in the sense of Mortini. In particular, N^p is a ring of Nevanlinna–Smirnov type (Theorem 1). We also give a necessary and sufficient condition for an ideal I in H^∞ to be the trace of an ideal J in N^p (Theorem 2). As an application, we give another sufficient condition for an ideal I in H^∞ to be trace of an ideal J in N^p, and in this case there holds $J = IN^p$ (Theorem 3). Theorem 4 gives a necessary and a sufficient condition for a prime ideal P in H^∞ to be the trace of some prime ideal Q in N^p.

In Section 3 we notice that N^p is a coherent ring for all $p > 1$, that is, the intersection of two finitely generated ideals in N^p is finitely generated (Theorem 5). Furthermore, the algebra N^p has the Corona Property (Theorem 6). We also give a sufficient condition for an ideal I of N^p, generated by a finite number of inner functions and which contains an interpolating Blaschke product B, to be equal to the whole space N^p (Theorem 7).

The basic result in Section 4 is a result of N. Mochizuki [26] which gives a complete characterization of the closed ideals of N^p (Theorem 8). A closed subspace E of N^p is invariant under multiplication by z if and only if it is an ideal (Theorem 9). Applying this result and a result of Mochizuki [26, Theorem 4], it can be proved that a closed subspace E of N^p is invariant if and only if it has the form φN^p for some inner function

φ (Theorem 10). This result is in fact the N^p-analogue of the famous Beurling's theorem for the Hardy spaces H^q $(0 < q < \infty)$.

2 THE IDEALS IN N^p AND H^∞

Following R. Mortini [27], we have the following definition.

Definition 1. A ring R satisfying $H^\infty \subset R \subset N$ is said to be of *Nevanlinna-Smirnov type* if every function $f \in R$ can be written in the form g/h, where g and h belong to the space H^∞ and h is an invertible element in R.

In particular, the Nevanlinna class N and the Smirnov class N^+ are rings of Nevanlinna-Smirnov type; hence the name (see [4, Chapter 2]). Further, Mortini noticed that by a result of M. Stoll [31], the ring $F^+ \cap N$ is of Nevanlinna-Smirnov type, where the space F^+ is the containing Fréchet envelope for N^+, consisting of those functions f holomorphic in $\mathbb{D}$ satisfying

$$\limsup_{r\to 1}(1-r)\log M(r,f) = 0$$

with $M(r,f) = \max_{|z|=r} |f(z)|$ (see [34]).

By Theorem A, it is easy to show the following result (see [6], where N^p is denoted as N_α^+).

Theorem C ([6]). *A function $f \in N$ belongs to the Privalov class N^p if and only if it can be expressed as the ratio g/h, where g and h are in H^∞, and h is an outer function such that* $\log|h^*| \in L^p(T)$.

Clearly, by Theorem B, every function h described in Theorem C is an invertible element of N^p. Therefore, we have the following result.

Theorem 1 ([14, Theorem B]). N^p $(1 < p < \infty)$ *is a ring of Nevanlinna–Smirnov type.*

As an application of Theorems A and B and the results of Mortini in [27], in Section 2 of [14] were obtained some facts about the ideal structure of the algebra N^p.

Definition 2. We say that an ideal I in H^∞ is the *trace* of an ideal J in N^p if $I = J \cap H^\infty$.

The following result is an immediate consequence of Theorems A, B and [27, Satz 1, Satz 2].

Theorem 2 ([14, Theorem 1]). *An ideal I in H^∞ is the trace of an ideal J in N^p if and only if the following condition is satisfied: If $f \in I$, F is an outer function with $\log|F^*| \in L^p(T)$, and if $fF \in H^\infty$, then $fF \in I$. In this case, J is a unique ideal in N^p with $I = J \cap H^\infty$, and there holds $J = IN^p$.*

Further, the above theorem immediately yields the following result.

Theorem 3 ([14, Theorem 2]). *Suppose that I is an ideal in H^∞ such that $f \in I$ implies that the inner factor of f also belongs to I. Then I is the trace of an ideal J in N^p, and there holds $J = IN^p$.*

Remark 2. As noticed in [14, p. 130, Remark], it remains an open question is it true the converse of Theorem 3. While this is true for the Nevanlinna class and the Smirnov class [27, Korrolar 1 and Korrolar 2, resp.], the corresponding problem is here complicated by the fact that there exist outer functions which are not invertible in N^p.

Definition 3. An ideal P in a ring R is *prime* if whenever $fg \in P$, $f, g \in R$, then either f or g is in P.

A characterization of the invertible elements in N^p and a result in [27, Satz 3] yield the following result established in [14].

Theorem 4 ([14, Theorem 3]). *A prime ideal P in H^∞ is the trace of some prime ideal Q in N^p if and only if P contains no outer functions F for which* $\log|F^*| \in L^p(T)$. *When this is the case, Q is a unique prime ideal in N^p with this property, and there holds $Q = PN^p$.*

Remark 3. By a result of Mochizuki [26, Theorem 3] (see [14, p. 131, Remark]), every prime ideal of N^p which is not dense in N^p is equal to the set of functions in N^p vanishing at a specific point of $\mathbb{D}$. The analogous result for the class N^+ was proved in [29, Theorem 1].

3 FINITELY GENERATED IDEALS IN N^p

Definition 4. An ideal J in the ring R such that $H^\infty \subset R \subset N$, is called *finitely generated* if there exist elements $f_1, \dots, f_n \in R$ such that

$$J = (f_1, \dots, f_n) = \left\{ \sum_{i=1}^{n} g_i f_i : g_i \in R \right\}.$$

If n can be chosen to be one, then J is a *principal ideal.* A ring R is said to be *coherent* if the intersection of two finitely generated ideals in R is finitely generated.

Using the result in [13] that H^∞ is a coherent ring, it was shown in [27, Satz 7] that this is true for all rings of Nevanlinna–Smirnov type. In particular, by Theorem 1, we have the following result.

Theorem 5 ([14, Theorem 4]). *N^p is a coherent ring for all $p > 1$.*

Definition 5. We say that a commutative ring R with unit of holomorphic functions on the disk $\mathbb{D}$ has the *Corona Property* if the ideal generated by $f_1, \dots, f_n \in R$ is equal to R if and only if there is an invertible element f of R such that

$$|f(z)| \leqslant \sum_{i=1}^{n} |f_i(z)| \quad \text{for all} \quad z \in \mathbb{D}.$$

Definition 5 is motivated by the famous Corona Theorem of Carleson (for example, see [7, p. 324] or [4, p. 202]), which states that the algebra H^∞ of all bounded holomorphic functions on $\mathbb{D}$ has the Corona Property. Mortini noticed [27, Satz 4] that by a result of

Wolff [7, p. 329], it is easy to show that every ring of Nevanlinna–Smirnov type has the Corona Property. In particular, by Theorem 1 we have the following result.

Theorem 6 ([14, Theorem 5]). *The algebra* N^p *has the Corona Property for all* $p > 1$.

Remark 4. It was proved in [11, Theorem 7] that there exists a subalgebra of the Nevanlinna class N containing the Smirnov class N^+ without the Corona Property.

Definition 6. A sequence $\{z_k\}_{k=1}^{\infty} \subset \mathbb{D}$ is called an *interpolating sequence* (for H^∞) if for every bounded sequence $\{\omega_k\}_{k=1}^{\infty}$ of complex numbers there exists a function f in H^∞ such that $f(z_k) = \omega_k$ for every $k = 1, 2, \ldots$. An *interpolating Blaschke product* is a Blaschke product given by (3) whose (simple) zeros form an interpolating sequence.

The following theorem given in [14] generalizes Theorem 6 in [27].

Theorem 7 ([14, Theorem 7]). *Assume that* I *is an ideal in* N^p *generated by inner functions* $\varphi_1, \ldots, \varphi_n$, *and suppose that* I *contains an interpolating Blaschke product* B *with zeros* $\{z_k\}_{k=1}^{\infty}$ *such that*

$$\sum_{k=1}^{\infty} \left(1 - |z_k|^2\right) |\log\left(|\varphi_1(z_k)| + \cdots + |\varphi_n(z_k)|\right)|^p < \infty.$$

Then $I = N^p$.

4 IDEALS IN THE SPACES N^p GENERATED BY INNER FUNCTIONS

Let U denote the operator of "multiplication by z" on the space N^p, that is,

$$(Uf)(z) = zf(z) \quad (f \in N^p, z \in \mathbb{D}).$$

U is called the *right shift* or *unilateral shift* because the Taylor coefficients of f one unit to the right.

Definition 7. An *invariant subspace* of the space N^p is defined as a closed subspace E of N^p such that $(Uf)(z) \in E$ whenever $f \in E$.

A characterization of the closed ideals of N^p is completely given by N. Mochizuki [26] as follows.

Theorem 8 ([26, Theorem 4]; cf. also see [22, Theorem 2.1]). *Let* $\mathcal{M}$ *be a closed ideal in* N^p *which is not identically* 0. *Then there is a unique modulo constants inner function* φ *defined by* (7) *such that* $\mathcal{M} = \varphi N^p$, *where*

$$\varphi N^p = \{\varphi f : f \in N^p\}.$$

The following result was attributed in [22].

Theorem 9 ([22, Lemma 2.2]). *A closed subspace* E *of* N^p *is invariant if and only if it is an ideal.*

As an immediate consequence of Theorems 8 and 9, it is obtained in [22] the following N^p-analogue of the famous Beurling's theorem for the Hardy spaces H^q ([1]; also see [9, Ch. 7, p. 99]).

Theorem 10 ([22, Theorem 2.3]; cf. also [20, the assertion 2.3 on p. 99]). *A closed subspace E of N^p is invariant if and only if it has the form φN^p for some inner function φ.*

Remark 5. Theorem 10 shows that there is a one-to-one correspondence between inner functions and invariant subspaces of N^p; so each invariant subspace of N^p being of the form of an ideal φN^p, where φ is an inner function.

Remark 6. By [29, Theorem 2], it follows that Theorem 8 is also true for the Smirnov class N^+.

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