MATHEMATICA MONTISNIGRI
Vol L (2021)

NOTE ON A THEOREM OF ZEHNXIAG ZHANG

I. LAIB[1], A. DERBAL[2], R. MECHIK[3] , N. REZZOUG[4*]

[1] ENSTP, Garidi Kouba, 16051, Algiers, and Laboratory of Equations with Partial Non Linear Derivatives, ENS Vieux Kouba, Algiers, Algeria.
[2] Department of Mathematics, Equation With Partial Non Linear Derivatives Laboratory, ENS Old Kouba, Algiers, Algeria.
[3] Department of Mathematics, University of Science and Technology Houari Boumédiène, Algiers, Algeria.
[4] University of Tiaret, and Analysis and Control of Partial Differential Equation Laboratory, University of Sidi Bel Abbes, Algeria.
*Corresponding author. E-mail: nadir793167115@gmail.com

DOI: 10.20948/mathmontis-2021-50-5

Summary. A sequence of strictly positive integers is said to be primitive if none of its terms divides the others. In this paper, we give a new proof of a result, conjectured by P. Erdős and Z. Zhang in 1993, on a primitive sequence whose the number of the prime factors of the termes counted with multiplicity is at most 4. The objective of this proof is to improve the complexity, which helps to prove this conjecture.

1. INTRODUCTION

A sequence A of strictly positive integers is said to be primitive if none of its terms divides the others. We define the degree of A by $deg(\mathrm{A}) = max\{\Omega(a)\ a \in \mathrm{A}\}$ where $\Omega(a)$ is the number of prime factors of a counted with multiplicity, we take $deg(A) = 0$ if $A = \{1\}$ or $\emptyset$. Erdős [2] showed that for a primitive set A, $\sum_{a\in A}\frac{1}{a\log a} < \infty$. Later in [3], Erdős asked if is true that for any primitive sequence A,

$$\sum_{a\in A,\ a\le n}\frac{1}{a\log a} \le \sum_{p\in P,\ p\le n}\frac{1}{p\log p}\ for\ n > 1,$$

where P denotes the set of prime numbers. After a few years, Zhang [5], proved the following:

Theorem. For any primitive sequence A whose the number of the prime factors of the termes counted with multiplicity is at most 4, we have

$$\sum_{a\in A,\ a\le n}\frac{1}{a\log a} \le \sum_{p\in P,\ p\le n}\frac{1}{p\log p}\ for\ n > 1.$$

In our work, by using the new estimations of the n-th prime number, we simplify the complexity (the number $N = 20000$ decreased to 95). Throughout the paper we denotes by p_m the m-th prime number and we put $f(A) = \sum_{a\in A}\frac{1}{a\log a}$ where, $f(A) = 0$ if $deg(A) = 0$.

2010 Mathematics Subject Classification: Primary 11Bxx.
Key words and Phrases: Primitive Sequence, Prime Number, Erdős Conjecture.