REMARKS ON DUDEK'S E-GROUPOIDS

SONIA DOG

Business School
Pervomayskaya str, 39600 Kremenchuk, Ukraine
e-mail: soniadog2@gmail.com

Summary. In this short note we prove that the axiom **EE**pq**EE**qr**E**rp cannot be used as a sole axiom for the equivalential calculus. It is a negative answer to the Dudek's problem posed in 1995.

The Polish logician S. Leśniewski, and some of his collaborators were interested in the extension, by the use of quantifiers and functorial variables, of versions of the propositional calculus in which the only undefined truth functor is **E**, "if and only if". Such calculus, introduced by Leśniewski in [7], is called the *equivalential calculus.*

In [8] J. Łukasiewicz proved that the equivalential calculus can be uniquely defined by each any one of the following axioms **EE**pq**EE**rq**E**pr, **EE**pq**EE**pr**E**rq and **EE**pq**EE**rp**E**qr. Other axiom systems and open problems one can find in [1], [2], [6] and [9].

To solve one of these problems W. A. Dudek introduced in 1995 the class of algebras (called *E-groupoids*) and proved (see also [4] and [5]) that this class can be used to the characterization of equivalential calculus and consequently to solve these problems.

In [3] he defined an E-groupoid as a non-empty set G with a binary operation denoted by juxtaposition and a distinguished element 0 such that the following axioms are satisfied:

(1) $xy \cdot (zx \cdot yz) = 0,$

(2) $(x \cdot yz)(xy \cdot z) = 0,$

(3) $x0 = 0 \quad \textit{implies} \quad x = 0.$

Next he proved in [3] and [4] that E-groupoids are strongly connected with BCK-algebras, BCI-algebras and BCC-algebras inspired by implicative logic and intensively studied by many authors.

Using the method of translations of logical theorems into algebraic identities (proposed in [5]) the above identities (1) and (2) can be written in the language of equivalential calculus as **EEE**pq**E**rp and **EE**p**E**qr**EE**pqr, respectively.

One of important problem on equivalential calculus is question on the possible axiomatization of this calculus by one of the following axioms: **EE**pq**EE**rp**E**rq, **EE**pq**EE**rq**E**rp, **EE**pq**EE**qr**E**pr, **EE**pq**EE**pr**E**qr, **EE**pq**EE**qr**E**rp.

From results obtained in [3] it follows that the first four from the above axioms cannot be used as a sole axiom. The problem of the last axiom is not solved in [3] and is left as open problem (see also [5]).

2010 Mathematics Subject Classification: 03G25, 20N02.
Key words and Phrases: Equivalential Calculus, Groupoid, Truth Functor.

Theorem. *The axiom* **EE**pq**EE**qr**E**rp *cannot be used as a sole axiom of the equivalential calculus.*
Proof. Note first that, used method proposed in [3], the above axiom can be translated into the identity
(4) $(xy \cdot yz) \cdot zx = 0.$

Simple calculations show that the set $G = \{0, a, b, c\}$ with the multiplication defined by table

$\cdot$	0	a	b	c
0	0	a	b	c
a	b	c	0	a
b	c	b	a	0
c	a	0	c	b

is a groupoid satisfying (4). In this groupoid $aa \cdot (aa \cdot aa) = c$ and $(c \cdot cc)(cc \cdot c) = a$. So, (1) and (2) are not satisfied. Hence a groupoid satisfying (4) may not be an E-groupoid. Therefore, the axiom **EE**pq**EE**qr**E**rp cannot be used as a sole axiom of the equivalential calculus.

REFERENCES

[1] Y. Arai, "On axiom systems of propositional calculi. XVII", *Proc. Japan Acad.* **42**, 351-354 (1966).
[2] Y. Arai and S. Tanaka, "On axiom systems of propositional calculi. XIX", *Proc. Japan Acad.* **42,** 358-360 (1966).
[3] W.A. Dudek, "Algebras connected with the equivalential calculus", *Math. Montisnigri*, **4,** 13-18 (1995).
[4] W.A. Dudek, "Algebras motivated by the equivalential calculus', *Rivista Mat. Pura et Appl,* **17**, 107-112 (1996).
[5] W.A. Dudek, "Algebras inspired by the equivalential calculus", *Italian J. Pure Appl. Math.* **9**, 139-148 (2001).
[6] K. Iseki, "On axiom systems of propositional calculi. XV", *Proc. Japan Acad.* **42**, 217-220 (1966).
[7] S. Leśniewski, "Grundruge eines neuen Systems der Grundlagen der Mathematik", *Fund. Math.* **14**, 1-81 (1929).
[8] B. Sobociński, "*An investigation of protothetic*", Editions de l'Institut d'Etudes Polonaises en Belgique, Brussels, (1949).
[9] S. Tanaka, "On axiom systems of propositional calculi. XVIII", *Proc. Japan Acad.* **42**, 355-357 (1966).
[10] S. Tanaka, "On axiom systems of propositional calculi. XX", *Proc. Japan Acad.* **42**, 61-363 (1966).

Received July 27, 2017