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ON A TOPOLOGICAL OBSERVATION CONNECTED WITH TOPOLOGY AND ⋆-TOPOLOGY

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Summary. Valuable collections such as semiopen, ⋆-open, semi*-I-open etc. have been handled in the most of the research areas of the topology for many applications. In the present paper, a new topological collection, namely $\text{semi}_{\text{I}}^{\star}$-open sets, containing topology and ⋆-topology is introduced and studied. Basic investigations of $\text{semi}_{\text{I}}^{\star}$-open subsets are discussed.

1 INTRODUCTION

In 1963, Levine introduced the collection of semiopen sets [18]. In 1990, Janković and Hamlett studied the concept of ⋆-topology [16]. In 2012, Ekici and Noiri introduced the notion of semi*-I-open sets [10]. After the basic papers, many papers related to main subjects have been occured in the topology and furthermore, these collections have been handled in the most of the research areas of the topology, [1]-[7], [12]-[14], [19,20], etc. In the present paper, a new topological collection, namely $\text{semi}_{\text{I}}^{\star}$-open sets, containing topology and ⋆-topology is introduced and studied. Main investigations of $\text{semi}_{\text{I}}^{\star}$-open sets are discussed.

A topological space will be denoted by $(\mathfrak{R},\lambda)$ and cl(R) and int(R) denote the closure and the interior of $R\subset\mathfrak{R}$, respectively, in this paper.

Suppose that I is a nonempty family of sets in a set $\mathfrak{R}$. Consider the following two conditions: (a) If $P\in\text{I}$ and $R\subset P$, $R\in\text{I}$, (b) If $R\in\text{I}$ and $P\in\text{I}$, $R\cup P\in\text{I}$. If these two conditions are satisfied, I is called an ideal on $\mathfrak{R}$ [17].

Suppose that $(\mathfrak{R},\lambda)$ is a topological space and I is an ideal on $\mathfrak{R}$. The following is called the local function of R (with respect to I and λ): $(.)^{\star}:P(\mathfrak{R})\rightarrow P(\mathfrak{R})$, $R^{\star}=\{l\in\mathfrak{R}: R\cap P\notin\text{I}$ for each $P\in\lambda$ such that $l\in P\}$ [17].

$cl^{\star}(R)=R\cup R^{\star}$ is a Kuratowski closure operator [16]. In [16], a new topology called ⋆-topology was introduced. It is generated by $cl^{\star}$ and denoted by $\lambda^{\star}$ [16].

Definition 1.1 A set R in an ideal space $(\mathfrak{R},\lambda,\text{I})$ is said to be

(a) β-I-open [15] if $R\subset cl(int(cl^{\star}(R)))$.
(b) semi-I-open [15] if $R\subset cl^{\star}(int(R))$.
(c) semi*-I-open [10] if $R\subset cl(int^{\star}(R))$.
(d) semi*-I-closed [10] if X\R is semi*-I-open.

Lemma 1.2 ([10]) Let $(\mathfrak{R},\lambda,\text{I})$ be an ideal space. Each semi-I-open set is semi*-I-open.

Definition 1.3 ([8]) A set R in an ideal space $(\mathfrak{R},\lambda,\text{I})$ is said to be

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(a) β_I^*-open if $R\subset cl(int^\star(cl(R)))$.
(b) β_I^*-closed if $X\backslash R$ is β_I^*-open.
(c) pre_I^*-open if $R\subset int^\star(cl(R))$.

Theorem 1.4 ([8]) Let $(\mathfrak{R},\lambda,I)$ be an ideal space and $R\subset\mathfrak{R}$. If R is a β-I-open set, then R is a β_I^*-open set.

A set R in a space $(\mathfrak{R},\lambda)$ is called semiopen [18] if $R\subset cl(int(R))$.

2 A NEW TOPOLOGICAL OBSERVATION CONNECTED WITH TOPOLOGY AND ⋆-TOPOLOGY: semi$_I^\star$-OPEN SETS

In this section, the collections of semi$_I^\star$-open sets and semi$_I^\star$-closed sets are introduced.

Definition 2.1 Let $(\mathfrak{R},\lambda,I)$ be an ideal space and $\Upsilon=\{\mathfrak{R}\}\cup\{P\subset\mathfrak{R}: P\neq\mathfrak{R}$ and there exists a ⋆-open set $E\neq\mathfrak{R}$ such that $P\backslash cl(E)\in I\}$.
(a) If $R\in\Upsilon$, then $R\subset\mathfrak{R}$ is called semi$_I^\star$-open.
(b) If $\mathfrak{R}\backslash R$ is semi$_I^\star$-open, $R\subset\mathfrak{R}$ is called semi$_I^\star$-closed.

Theorem 2.2 Let $(\mathfrak{R},\lambda,I)$ be an ideal space and $R_\alpha\subset\mathfrak{R}$ be semi$_I^\star$-open sets for $\alpha\in\lambda$. Then $\cap_{\alpha\in\lambda}R_\alpha\subset\mathfrak{R}$ is semi$_I^\star$-open.

Proof. In case $R_\alpha=\mathfrak{R}$ for each $\alpha\in\lambda$, it follows that $\cap_{\alpha\in\lambda}R_\alpha\subset\mathfrak{R}$ is semi$_I^\star$-open.
Assume $R_{\alpha_\beta}\neq\mathfrak{R}$ for an $\alpha_\beta\in\lambda$. Since $R_{\alpha_\beta}\neq\mathfrak{R}$ is semi$_I^\star$-open in $\mathfrak{R}$, then there exists a ⋆-open set $F\neq\mathfrak{R}$ such that $R_{\alpha_\beta}\backslash cl(F)\in I$. Since $\cap_{\alpha\in\lambda}R_\alpha\subset R_{\alpha_\beta}$, then $(\cap_{\alpha\in\lambda}R_\alpha)\backslash cl(F)\subset R_{\alpha_\beta}\backslash cl(F)\in I$. This implies $(\cap_{\alpha\in\lambda}R_\alpha)\backslash cl(F)\in I$. Therefore, $\cap_{\alpha\in\lambda}R_\alpha\subset\mathfrak{R}$ is semi$_I^\star$-open.

Remark 2.3 There exist semi$_I^\star$-open sets $R\subset\mathfrak{R}$ and $P\subset\mathfrak{R}$ such that $R\cup P$ is not semi$_I^\star$-open for an ideal space $(\mathfrak{R},\lambda,I)$.

Example 2.4 Let consider the ideal space $(\mathfrak{R},\lambda,I)$ where $\mathfrak{R}=\{\omega, \phi, \nu, \varrho, \iota\}$, $\lambda=\{\mathfrak{R}, \{\omega, \phi\}, \{\nu, \varrho, \iota\}, \varnothing\}$ and $I=\{\varnothing\}$. In this ideal space, $R=\{\omega\}\subset\mathfrak{R}$ and $P=\{\nu\}\subset\mathfrak{R}$ are semi$_I^\star$-open but $R\cup P$ is not semi$_I^\star$-open.

Theorem 2.5 Let $(\mathfrak{R},\lambda,I)$ be an ideal space. Assume that $R\neq\mathfrak{R}$ is semi$_I^\star$-open and $P\subset R$. Then P is semi$_I^\star$-open in $\mathfrak{R}$.

Proof. Assume that $R\neq\mathfrak{R}$ is semi$_I^\star$-open and $P\subset R$. Then there exists a ⋆-open set $F\neq\mathfrak{R}$ such that $R\backslash cl(F)\in I$. Moreover, we have

$$P\backslash cl(F)\subset R\backslash cl(F)\in I.$$

Therefore, $P\backslash cl(F)\in I$. Thus, $P\subset\Re$ is $semi_I^\star$-open in $\Re$.

Theorem 2.6 Let $(\Re,\lambda,I)$ be an ideal space. Assume that $R\subset\Re$ is β_I^*-open and also it is not dense in $\Re$. Then R is $semi_I^\star$-open in $\Re$.

Proof. Let $R\subset\Re$ be β_I^*-open set. Let R be not dense in $\Re$. It follows $R\subset cl(int^\star(cl(R)))$. Moreover, $cl(R)\neq \Re$ and thus

$$int^\star(cl(R))\neq \Re.$$

Put $E=int^\star(cl(R))$. Therefore, we have

$$R\backslash cl(E)\in I.$$

Hence, $R\subset\Re$ is $semi_I^\star$-open.

Remark 2.7 Let $(\Re,\lambda,I)$ be an ideal space. There exists a β_I^*-open subset such that it is not $semi_I^\star$-open in $\Re$ as it is shown in the following example.

Example 2.8 Let consider the ideal space $(\Re,\lambda,I)$ where $\Re=\{\omega, \phi, \nu, \varrho, \iota\}$, $\lambda=\{\Re, \{\omega, \phi\}, \{\nu, \varrho, \iota\}, \varnothing\}$ and $I=\{\varnothing\}$. In this ideal space, $R=\{\omega, \nu\}\subset\Re$ is β_I^*-open but R is not $semi_I^\star$-open.

Remark 2.9 Let $(\Re,\lambda,I)$ be an ideal space. The following example shows that there exists a $semi_I^\star$-open subset such that it is not β_I^*-open in $\Re$.

Example 2.10 Let consider the ideal space $(\Re,\lambda,I)$ where $\Re=\{\omega, \phi, \nu, \varrho\}$, $\lambda=\{\Re, \{\omega\}, \{\phi, \nu\}, \{\omega, \phi, \nu\}, \varnothing\}$ and $I=\{\varnothing, \{\omega\}, \{\varrho\}, \{\omega, \varrho\}\}$. In this ideal space, $R=\{\varrho\}\subset\Re$ is $semi_I^\star$-open but R is not β_I^*-open.

Corollary 2.11 Let $(\Re,\lambda,I)$ be an ideal space. Assume that $R\subset\Re$ is β-I-open or pre_I^*-open and also it is not dense in $\Re$. Then R is $semi_I^\star$-open in $\Re$.

Proof. Let $R\subset\Re$ be β-I-open or pre_I^*-open. Then it is β_I^*-open by Theorem 2.2 of [8]. So, it follows from Theorem 2.6.

Theorem 2.12 Let $(\Re,\lambda,I)$ be an ideal space. $R\subset\Re$ is $semi_I^\star$-open if and only if $R=\Re$ or there exist $F\in I$ and a $\star$-open set $J\neq\Re$ such that $R\backslash F\subset cl(J)$.

Proof. Let $R\subset\Re$ be $semi_I^\star$-open. Assume that $R\neq\Re$. Since $R\subset\Re$ is $semi_I^\star$-open, this implies that there exists a $\star$-open set $J\neq\Re$ such that $R\backslash cl(J)\in I$. We have

$$R\backslash(R\backslash cl(J))\subset cl(J).$$

Let $F=R\backslash cl(J)$. Consequently, $R\backslash F\subset cl(J)$.

Conversely, assume $R=\Re$. Then $R\subset\Re$ is $\text{semi}_I^\star$-open. Suppose that there exist $F\in I$ and a $\star$-open set $J\neq\Re$ such that $R\setminus F\subset cl(J)$. This implies that

$$R\setminus cl(J)\subset F.$$

Since $R\setminus cl(J)\subset F$, then $R\setminus cl(J)\in I$. Hence, $R\subset\Re$ is $\text{semi}_I^\star$-open.

Theorem 2.13 Let $(\Re,\lambda,I)$ be an ideal space. $R\subset\Re$ is $\text{semi}_I^\star$-closed iff $R=\emptyset$ or there exist $F\in I$ and a $\star$-closed set $E\neq\emptyset$ such that $int(E)\setminus F\subset R$.

Proof. Let $R\subset\Re$ be $\text{semi}_I^\star$-closed. Let consider $R\neq\emptyset$. Then we have $\Re\setminus R\neq\Re$. Since $\Re\setminus R\subset\Re$ is $\text{semi}_I^\star$-open, then there exists a $\star$-open subset $J\neq\Re$ such that

$$(\Re\setminus R)\setminus cl(J)\in I.$$

Let $F=(\Re\setminus R)\setminus cl(J)$. Then $F\in I$ and $\Re\setminus R\subset cl(J)\cup F$. It follows that

$$(\Re\setminus cl(J))\cap(\Re\setminus F)\subset R.$$

Let consider $E=\Re\setminus J$. Then $E\subset\Re$ is $\star$-closed and $E\neq\emptyset$. Furthermore, $int(E)\cap(\Re\setminus F)\subset R$. Hence, $int(E)\setminus F\subset R$.

Conversely, let $R=\emptyset$. Then R is $\text{semi}_I^\star$-closed in $\Re$. Suppose that there exist $F\in I$ and a $\star$-closed set $E\neq\emptyset$ such that

$$int(E)\setminus F\subset R.$$

We have $\Re\setminus R\subset(\Re\setminus int(E))\cup F$. Let $J=\Re\setminus E$. Then $J\neq\Re$ is $\star$-open. Since

$$\Re\setminus R\subset(\Re\setminus int(E))\cup F,$$

then we have $\Re\setminus R\subset cl(J)\cup F$. So, $(\Re\setminus R)\setminus F\subset cl(J)$. Concequently, $\Re\setminus R\subset\Re$ is $\text{semi}_I^\star$-open in $\Re$ by Theorem 2.12 and hence $R\subset\Re$ is $\text{semi}_I^\star$-closed in $\Re$.

Theorem 2.14 Let $(\Re,\lambda,I)$ be an ideal space. $R\subset\Re$ is $\text{semi}_I^\star$-closed iff $R=\emptyset$ or there exists a $\star$-closed subset $E\neq\emptyset$ such that $int(E)\setminus R\in I$.

Proof. Let $R\subset\Re$ be $\text{semi}_I^\star$-closed. By Theorem 2.13, there exist $F\in I$ and a $\star$-closed set $E\neq\emptyset$ such that $int(E)\setminus F\subset R$. It follows that $int(E)\setminus R\subset F$. Therefore, $int(E)\setminus R\in I$.

Conversely, in case $R=\emptyset$, it follows that $R\subset\Re$ is $\text{semi}_I^\star$-closed. Assume that there exist a $\star$-closed set $E\neq\emptyset$ such that $int(E)\setminus R\in I$. Let consider $F=int(E)\setminus R$. Then F is in I and $int(E)\setminus F\subset R$. Thus, by Theorem 2.13, $R\subset\Re$ is $\text{semi}_I^\star$-closed.

3 FURTHER INVESTIGATIONS

In this section, further investigations on the collections of semi$_{I}^{\star}$-open sets and semi$_{I}^{\star}$-closed sets are discussed.

Theorem 3.1 Let $(\mathfrak{R},\lambda,I)$ be an ideal space. Suppose that $R\neq\mathfrak{R}$, $int^{\star}(R)=\emptyset$ and $R\in I$. Then $R\subset\mathfrak{R}$ is semi$_{I}^{\star}$-open.

Proof. Let $R\neq\mathfrak{R}$, $int^{\star}(R)=\emptyset$ and $R\in I$. Then

$$R\setminus cl(int^{\star}(R)))\in I.$$

Thus, $R\subset\mathfrak{R}$ is semi$_{I}^{\star}$-open.

Theorem 3.2 Let $(\mathfrak{R},\lambda,I)$ be an ideal space. If there exists a $\star$-open set P such that $P\neq\mathfrak{R}$ and P is dense, each set in $\mathfrak{R}$ is semi$_{I}^{\star}$-open.

Proof. Let $P\subset\mathfrak{R}$ be $\star$-open and dense such that $P\neq\mathfrak{R}$. Let $R\subset\mathfrak{R}$. In case $R=\mathfrak{R}$, it follows that R is semi$_{I}^{\star}$-open. In case $R\neq\mathfrak{R}$, it follows that $R\setminus cl(P)\in I$. Therefore, R is semi$_{I}^{\star}$-open.

Definition 3.3 Let $(\mathfrak{R},\lambda,I)$, $(\aleph,\psi,J)$ be ideal spaces and $\ell: \mathfrak{R}\to\aleph$ be a function.

(a) If $\ell(R)$ is $\star$-closed for each $\star$-closed set R in $\mathfrak{R}$, $\ell: \mathfrak{R}\to\aleph$ is said to be $\star$-closed [9],

(b) If $\ell(R)$ is $\star$-open for each $\star$-open set R in $\mathfrak{R}$, $\ell: \mathfrak{R}\to\aleph$ is said to be $\star$-open [11].

Theorem 3.4 Let $(\mathfrak{R},\lambda,I)$, $(\aleph,\psi,\ell(I))$ be ideal spaces and $\ell: \mathfrak{R}\to\aleph$ be a bijection, $\star$-open and continuous function where $\ell(I)=\{\ell(I): I\in I\}$. $\ell(R)\subset\aleph$ is a semi$_{I}^{\star}$-open set for a semi$_{I}^{\star}$-open subset R.

Proof. Let $R\subset\mathfrak{R}$ be a semi$_{I}^{\star}$-open subset. In case $R=\mathfrak{R}$, it follows that $\ell(R)\subset\aleph$ is semi$_{I}^{\star}$-open in $\aleph$. Assume that $R\neq\mathfrak{R}$. Since $R\subset\mathfrak{R}$ is semi$_{I}^{\star}$-open, there exists a $\star$-open set $F\neq\mathfrak{R}$ such that $R\setminus cl(F)\in I$. We have $\ell(R\setminus cl(F))\in\ell(I)$. Then $\ell(R)\setminus\ell(cl(F))\in\ell(I)$. This implies

$$\ell(R)\setminus cl(\ell(F))$$
$$\subset \ell(R)\setminus\ell(cl(F))\in\ell(I).$$

and furthermore $\ell(R)\setminus cl(\ell(F))\in\ell(I)$. Since $\ell(R)\setminus cl(\ell(F))\in\ell(I)$, $\ell(R)\subset\aleph$ is semi$_{I}^{\star}$-open.

Corollary 3.5 Let $(\mathfrak{R},\lambda,I)$, $(\aleph,\psi,\ell(I))$ be ideal spaces and $\ell: \mathfrak{R}\to\aleph$ be a bijection, $\star$-closed and continuous function where $\ell(I)=\{\ell(I): I\in I\}$. $\ell(R)\subset \aleph$ is a semi$_{I}^{\star}$-open subset for semi$_{I}^{\star}$-open R.

Proof. Follows in view of Theorem 3.4.

Theorem 3.6 Let $(\Re,\lambda,I)$ be an ideal space. $R\subset\Re$ is semi$^\star_I$-closed in $\Re$ iff $R=\varnothing$ or there exist $F\in I$ and a $\star$-closed set $E\neq\varnothing$ such that $int(E)\subset R\cup F$.

Proof. Let $R\subset\Re$ be semi$^\star_I$-closed in $\Re$. By Theorem 2.14, $R=\varnothing$ or there exists a $\star$-closed subset $E\neq\varnothing$ such that

$$int(E)\backslash R\in I.$$

Put $F=int(E)\backslash R$. This implies $F\in I$ and furthermore, $int(E)\subset R\cup F$.

Conversely suppose that $R=\varnothing$ or there exist $F\in I$ and a $\star$-closed subset $E\neq\varnothing$ such that $int(E)\subset R\cup F$. This implies

$$int(E)\backslash R\subset F.$$

Since $F\in I$, then $int(E)\backslash R\in I$. Concequently, by Theorem 2.14, R is semi$^\star_I$-closed in $\Re$.

Theorem 3.7 Let $(\Re,\lambda,I)$ be an ideal space and let $\Psi=\{\Re\}\cup\{P\subset\Re$: $P\neq\Re$ and there exist a $\star$-open subset $F\neq\Re$ and an element G of I such that $P\subset cl(F)\cup G\}$. Then $R\subset\Re$ is semi$^\star_I$-open iff $R\in\Psi$.

Proof. Follows in view of Theorem 2.12.

Theorem 3.8 Let $(\Re,\lambda,I)$ be an ideal space and let $R\subset\Re$ be semi*-I-open. Then R is semi$^\star_I$-open in $\Re$.

Proof. Let $R\subset\Re$ be semi*-I-open in $\Re$. In case $R=\Re$, it follows that R is semi$^\star_I$-open in $\Re$. Let $R\neq\Re$. Since R is semi*-I-open in $\Re$, then $R\subset cl(int^\star(R))$. Assume $F=int^\star(R)$. This implies that F is $\star$-open in $\Re$. Also, we have $F\neq\Re$ and $R\backslash cl(F)\in I$. Consequently, R is semi$^\star_I$-open in $\Re$.

Corollary 3.9 Let $(\Re,\lambda,I)$ be an ideal space. Suppose that $R\subset\Re$ is semiopen or $\star$-open in $\Re$. Then R is semi$^\star_I$-open in $\Re$.

Proof. Assume that $R\subset\Re$ is semiopen or $\star$-open in $\Re$. This implies that R is semi*-I-open in $\Re$. Thus, by Theorem 3.8, R is semi$^\star_I$-open in $\Re$.

Remark 3.10 Let $(\Re,\lambda,I)$ be an ideal space. The following implications hold for $R\subset\Re$:

$$\begin{tikzcd}
\text{semiopen} \arrow[r] & \text{semi*-I-open} \arrow[r] & \text{semi}^\star_I\text{-open} \\
\text{semi-I-open} \arrow[u] & & \\
\text{open} \arrow[u] \arrow[r] & \star\text{-open} \arrow[uu] &
\end{tikzcd}$$

Remark 3.11 The above implications are irreversible as shown in the below examples. The related papers have the other examples.

Example 3.12 Let consider the ideal space $(\Re,\lambda,\mathrm{I})$ where $\Re=\{\omega, \phi, \nu, \varrho\}$, $\lambda=\{\Re, \{\omega\}, \{\omega, \phi\}, \{\nu, \varrho\}, \{\omega, \nu, \varrho\}, \varnothing\}$ and $\mathrm{I}=\{\varnothing, \{\omega\}, \{\varrho\}, \{\omega, \varrho\}\}$. In this ideal space, $R=\{\phi, \varrho\}\subset\Re$ is semi$_{\mathrm{I}}^{\star}$-open but R is not semi*-I-open. Meanwhile, $\{\phi\}$ is semi*-I-open but it is not semiopen.

Example 3.13 Let consider the ideal space $(\Re,\lambda,\mathrm{I})$ where $\Re=\{\omega, \phi, \nu, \varrho\}$, $\lambda=\{\Re, \{\omega\}, \{\phi, \nu\}, \{\omega, \phi, \nu\}, \varnothing\}$ and $\mathrm{I}=\{\varnothing, \{\omega\}, \{\varrho\}, \{\omega, \varrho\}\}$. In this ideal space, $R=\{\omega, \varrho\}\subset\Re$ is semi*-I-open but it is not $\star$-open.

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