

MATHEMATICAL MODELING OF THE TEMPERATURE STATE OF A PLANE LAYER POLYMER DIELECTRIC AT CONSTANT VOLTAGE

G. N. KUVYRKIN¹, I.Y. SAVELYEVA¹ AND V.S. ZARUBIN¹

¹ Bauman Moscow State Technical University (BMSTU)
2nd Baumanskaya st., 5, 105005 Moscow, Russian Federation
e-mail: inga.savelyeva@gmail.com

DOI: 10.20948/mathmon-2019-44-8

Summary. The use of modern polymeric materials as dielectrics makes it possible to increase operational characteristics of the elements of various electric power and electronic devices. The necessary combination of interconnected values of permissible electric field strength and maximum operating temperature is crucial when choosing a particular polymer material. The relationship of these parameters is nonlinear due to the nonlinear dependence the electrical resistivity and the thermal conductivity coefficient on temperature of the polymer material. This relationship can be represented in a closed analytical form of integral relations in which the variable limit of the integrals is the desired function describing the temperature distribution in the dielectric layer. A quantitative analysis of these ratios for five different polymeric materials with known electrothermal characteristics has been carried out. The results of calculations of the temperature state of the dielectric layer and the distribution in the layer of the absolute value of the electric field intensity are given. The presented results can be used to justify the choice of a particular polymer material as a dielectric in the designed devices.

1 INTRODUCTION

Existing polymeric materials used in various electrical and radio engineering devices as dielectrics have a fairly high electrical resistivity, characterized at a temperature of about 300 K by the values of $10^{14} \dots 10^{18}$ Ohm·m [1-3]. But with a significant difference in the potentials of the electric field on the surfaces of the dielectric layer, even at such electrical resistance values, the current passing through the layer causes the release of Joule heat, which leads to an increase in the dielectric temperature. The increase in temperature, as a consequence, leads to a decrease in the electrical resistivity, which in turn leads to a further increase in the amperage and volumetric energy release. The comparatively low coefficient of the thermal conductivity of polymeric materials [1, 4, 5] and the insufficient intensity of the heat dissipation of the released energy into the external environment create the prerequisites for a positive feedback, due to which there is a rapid increase in temperature terminating in thermal destruction of the dielectric material (melting, carburization). Such process was called the thermal breakdown of the dielectric, in contrast to the electric breakdown [6, 7].

The reliable functioning of the polymer dielectric with a high electrical potential difference is facilitated by the intensification of the escape of the Joule heat released in it and the choice of material with the highest possible value of the thermal conductivity, which is desirable to

2010 Mathematics Subject Classification: 00A00, 00B00, 00C00.

Key words and Phrases: Polymer Dielectric, One-dimensional Temperature Distribution, Volumetric Energy Release, Integral Relationships.

increase with increasing temperature, which is characteristic for some polymer materials. A quantitative analysis of the dielectric temperature state, which is necessary for evaluating its operability, requires the involvement of modern methods of the mathematical modeling [8-12] and is related to the solution of a rather complex nonlinear problem that takes into account the interrelationship of the dependences on the electrical resistivity of the dielectric and its thermal conductivity.

In this paper, the problem of steady-state thermal conductivity with a constant difference of electric potentials on these surfaces is formulated for a plane layer of a polymer dielectric with a given temperature of the cooled surface and an ideally insulated opposite surface. This problem forms the basis of the differential form of the mathematical model describing the temperature state of the dielectric layer. Necessary for the quantitative analysis of the temperature state of this layer the information about dependences on the electrical resistance temperature and the thermal conductivity coefficient for several polymeric materials used as dielectrics are presented. The differential form of the model is converted to two integral relations are used for calculation and subsequent comparison of the temperature distributions and the absolute values of the electric field in the layer made from different polymeric materials.

2 PROBLEM STATEMENT

Let a flat layer of polymer dielectric has a constant thickness h . Then, in the case of homogeneous conditions of heat exchange on each of two flat surfaces bounding this layer the steady-state temperature distribution $T(z)$ is one dimensional and is depending only on one coordinate z , counting in the direction normal to the surfaces of the layer. This distribution will satisfy a nonlinear differential equation [13]:

$$\frac{d}{d\zeta} \left(\lambda(T) \frac{dT(\zeta)}{d\zeta} \right) + q_V(T, \zeta) h^2 = 0, \quad (1)$$

where $\zeta = z/h$, λ – temperature-dependent thermal conductivity of the dielectric, q_V – volumetric power of energy release, caused by the transition of some of the electrical energy into Joule heat and depending both on temperature and on the coordinate. The thermal conductivity coefficient of polymer dielectrics, as a rule, increases with increasing temperature and depends significantly on their microstructure, which, in turn, can vary with temperature. In the case of an amorphous structure corresponding to the chaotic arrangement of polymer macromolecules, the thermal conductivity coefficient is usually the smallest and grows as the arrangement of the macromolecules is ordered, due to a certain increase in the polymer density and characterized by the degree of its crystallinity [1, 4].

In Fig. 1 for some polymeric materials that are used as dielectrics, temperature-dependent boundaries of the change in the coefficient of thermal conductivity upon transition from an amorphous structure (the lower boundary) to a structure with the greatest density (upper bound) are presented [1, 4]. For polipiromellitimide (position 1), the given curves are not related to the structural features of the polymer and refer to different modifications of the material produced by DuPont (USA) with the Capton trademark (the lower curve is Kapton H, and the upper one is Kapton HN [14]). Other materials also have different industrial brands

along with the names defined by the chemical composition: for example, 2 - dacron, lavsan, mylar, 3 - fluoroplast-3, 4 - macrolon, merlon, 5 - fluoroplast-4, teflon.

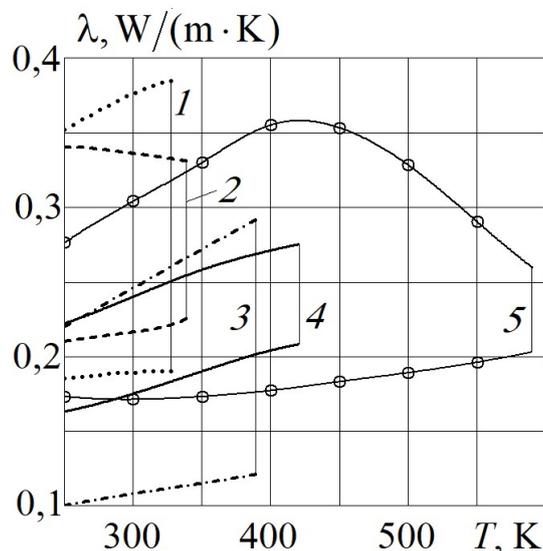


Figure 1: The upper and lower boundaries of the change with the temperature of the thermal conductivity for some polymer materials: 1 – polypyromellitimide; 2 – polyethylene terephthalate; 3 – polychlorotrifluoroethylene; 4 – polycarbonate; 5 – polytetrafluoroethylene

In Fig. 2 the experimentally obtained dependences on the temperature of the thermal conductivity of polytetrafluoroethylene of different densities γ are given [1, 15] (from the value of $\gamma_* = 2000 \text{ kg/m}^3$ corresponding to the amorphous structure to the highest value of $\gamma^* = 2300 \text{ kg/m}^3$). For intermediate values of the polymer dielectric density, its thermal conductivity at the fixed temperature is recommended to be estimated by the formula [4] $\lambda = \lambda_* + (\lambda^* - \lambda_*)(\gamma - \gamma_*) / (\gamma^* - \gamma_0)$, where λ_* and λ^* – the lower and the upper values of this coefficient at the density of the polymer γ_* and γ^* , accordingly. The results of calculations using this formula, shown in Fig. 2 for comparison, are in good agreement with the experimental data for the density values equal to (in kg/m^3) 2050 and 2250, the agreement for the density $\gamma^* = 2100 \text{ kg/m}^3$ was less reached, and in the remaining cases for some values of the temperature the deviations from the experimental data turned out to be more essential.

The power of volumetric energy release in a dielectric at a constant voltage is [16]

$$q_v(T, \zeta) = (E(\zeta))^2 / \rho(T), \quad (2)$$

where $E \geq 0$ – modulus of the electric field strength vector, ρ – temperature-dependent electrical resistance of a dielectric at constant voltage. In Fig. 3 for all five of the above mentioned polymeric materials, the dependences on the temperature of the decimal logarithm of the ratio ρ / ρ_* are presented using the data from [1], where, ρ_* , Ohm m is the value of the electrical resistivity at the temperature $T = 300 \text{ K}$ for each of the materials: 10^{16} – polypyromellitimide, $1,58 \cdot 10^{15}$ – polyethylene terephthalate, $6,31 \cdot 10^{16}$ –

polychlorotrifluoroethylene, $3,16 \cdot 10^{14}$ – polycarbonate, 10^{17} – polytetrafluoroethylene.

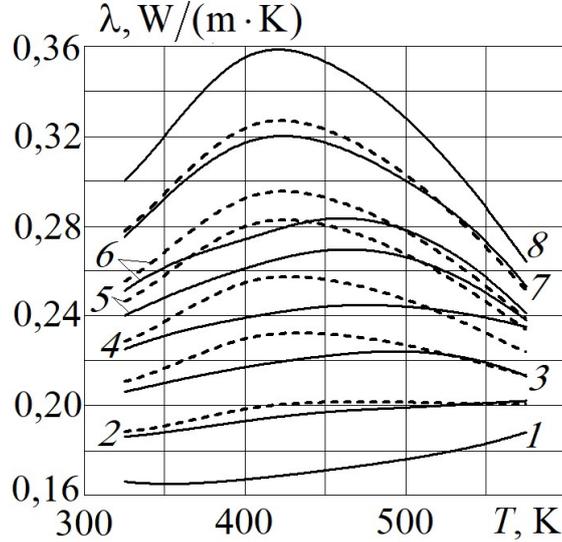


Figure 2: Temperature dependence of the thermal conductivity of polytetrafluoroethylene at different density values (in kg / m^3): 1 – 2000; 2 – 2050; 3 – 2100; 4 – 2140; 5 – 2180; 6 – 2200; 7 – 2250; 8 – 2300

At a constant voltage, the absolute value of the current density passing through the fragment of the dielectric layer is

$$j = E(\zeta) / \rho(T) = \text{const.} \quad (3)$$

Taking into account formulas (2) and (3) equation (1) has the form

$$\frac{d}{d\zeta} \left(\lambda(T) \frac{dT(\zeta)}{d\zeta} \right) + (jh)^2 \rho(T) = 0. \quad (4)$$

The surface of the dielectric layer with a coordinate $\zeta = 0$ will take is insulated and on the opposite cooled surface of the layer where $\zeta = 1$ we set a temperature T_1 , i.e.

$$\lambda(T) \frac{dT(\zeta)}{d\zeta} \Big|_{\zeta=0} = 0, \quad T(1) = T_1. \quad (5)$$

The electric field potential at $\zeta = 0$ put equal to zero, and the absolute value of the potential on the opposite surface denote by U_1 . It is clear that the formulated conditions are applicable to the dielectric layer doubled thickness with an absolute value of the potential difference $2U_1$ and given temperature values T_1 on both surfaces of such layer. Because of the nonlinearity of equation (4) its solution with boundary conditions (5) cannot be represented in an analytical form solved with respect to function $T(\zeta)$. However, with known dependences of $\lambda(T)$ and $\rho(T)$, it is possible to construct integral relations connecting these dependences with the desired temperature distribution in the dielectric layer. This temperature distribution is the variable limit of the integrals in these relations. Such a procedure for

solving the problem (in contrast to the finite-difference and finite-element methods) allows one to obtain the necessary numerical information with controlled accuracy, since well-known quadrature formulas are applicable for calculating the integrals.

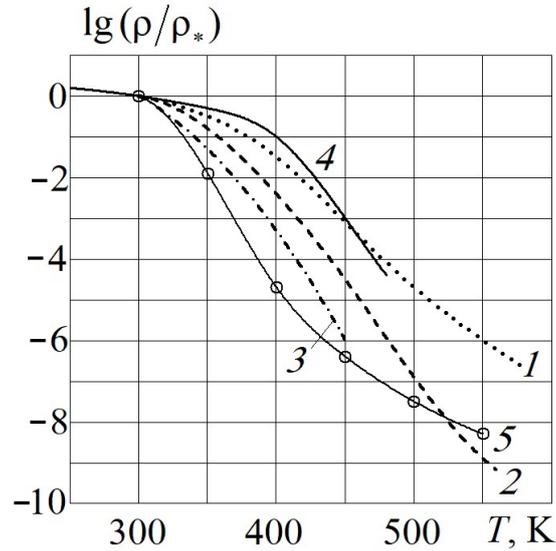


Figure 3: Dependence on the temperature of the electrical resistance for some polymeric materials (the designations of the curves are identical with Fig. 1)

3 INTEGRAL RELATIONS

The substitution $\lambda(T)dT(\zeta)/d\zeta = p$ allows to represent equation (4) in the form:

$$p \frac{dp}{dT} + (jh)^2 \lambda(T) \rho(T) = 0$$

and then write the first integral for this equation

$$\left(\lambda(T) \frac{dT(\zeta)}{d\zeta} \right)^2 = C_1 - 2(jh)^2 \int_{T_1}^T \lambda(T') \rho(T') dT'.$$

Denoting T_0 the unknown temperature value on the ideally insulated surface of the dielectric layer and using the boundary condition on this surface in the form of the first equality (5), we find the constant

$$C_1 = 2(jh)^2 \int_{T_1}^{T_0} \lambda(T') \rho(T') dT'$$

and after substitution C_1 in the first integral we obtain an ordinary differential equation of the first order

$$\lambda(T) \frac{dT(\zeta)}{d\zeta} = \pm jh \left(2 \int_{T_1}^{T_0} \lambda(T') \rho(T') dT' \right)^{1/2}. \quad (6)$$

Proceeding from the physical meaning of the problem, the left-hand side of equation (6) is nonpositive [13]. Therefore, on the right-hand side of this equation, the minus sign should be chosen and, after integration, written

$$\zeta = C_2 - \frac{1}{jh} \int_{T_0}^T \left(2 \int_{T'}^{T_0} \lambda(T'') \rho(T'') dT'' \right)^{-1/2} \lambda(T') dT'.$$

Using the second equality in (5), we find

$$C_2 = 1 + \frac{1}{jh} \int_{T_0}^{T_1} \left(2 \int_T^{T_0} \lambda(T') \rho(T') dT' \right)^{-1/2} \lambda(T) dT$$

and in the end we obtain the relation

$$1 - \zeta = \frac{1}{jh} \int_{T_1}^T \left(2 \int_{T'}^{T_0} \lambda(T'') \rho(T'') dT'' \right)^{-1/2} \lambda(T') dT'. \quad (7)$$

Equation (7) includes unknown values of j and T_0 . In the considered dielectric layer, the losses per unit time of electrical energy in the form of Joule heat per unit surface area of the layer are equal jU_1 . Therefore, by taking into account the formula (6), we can write

$$-\lambda(T) \frac{dT(\zeta)}{d\zeta} \Big|_{\zeta=1} = jh \left(2 \int_{T_1}^{T_0} \lambda(T) \rho(T) dT \right)^{1/2} = jhU_1.$$

Hence the integral relation follows

$$U_1^2 = 2 \int_{T_1}^{T_0} \lambda(T) \rho(T) dT, \quad (8)$$

which establishes a connection between the functions $\lambda(T)$ and $\rho(T)$ and the required temperature T_0 with a value U_1 .

After calculating from the relation (8) the values T_0 , putting in the formula (7) $\zeta = 0$ and $T = T_0$, we can find the value of the product jh :

$$jh = \int_{T_1}^{T_0} \left(2 \int_T^{T_0} \lambda(T') \rho(T') dT' \right)^{-1/2} \lambda(T) dT = I(T_0) \quad (9)$$

and then obtain an integral relation

$$1 - \zeta = \frac{1}{I(T_0)} \int_{T_1}^{T(\zeta)} \left(2 \int_{T'}^{T_0} \lambda(T'') \rho(T'') dT'' \right)^{-1/2} \lambda(T') dT', \quad (10)$$

which determines the temperature distribution over the thickness of the dielectric layer.

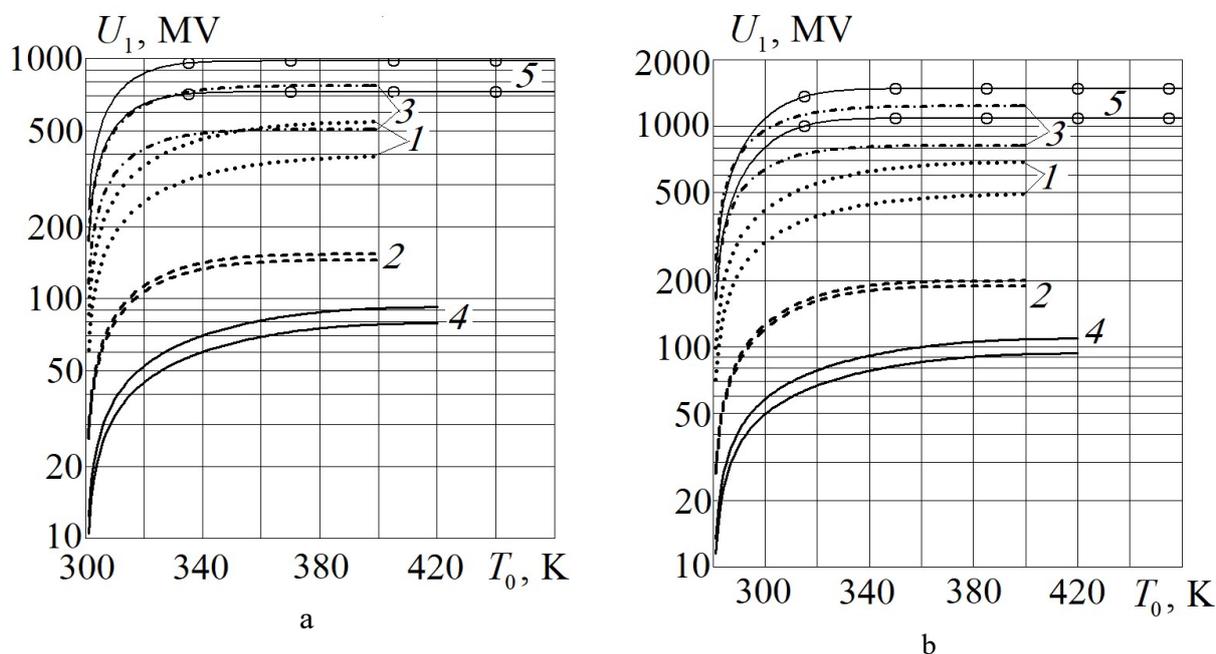


Figure 4: The relationship between the temperature of the ideally insulated surface of the dielectric layer from various polymer materials and the electrical potential at a given value T_l – the temperature of the cooled surface of the layer (the designations of the curves are identical with Fig. 1);
 a – $T_l = 300$ K; b – $T_l = 280$ K

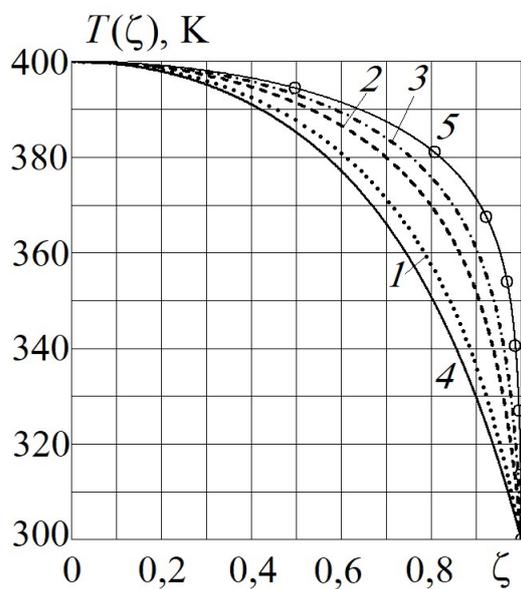


Figure 5: The temperature distribution over the thickness of the dielectric layer from various polymer materials (the designations of the curves are identical with Fig. 1)

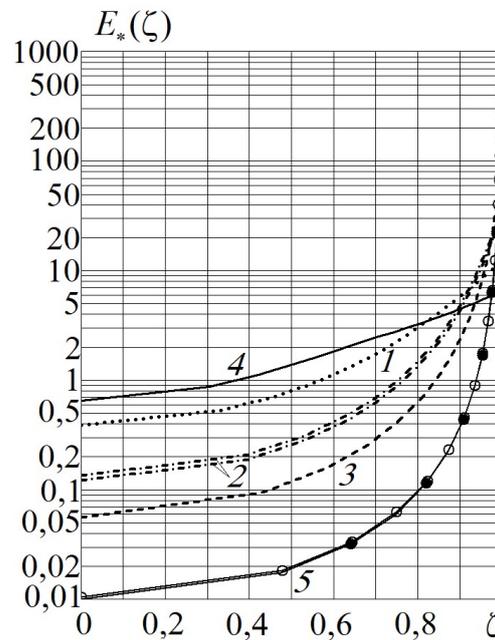


Figure 6: The distribution of the absolute value of the electric field strength along the thickness for various polymeric materials (the designations of the curves are identical with Fig. 1)

4 QUANTITATIVE ANALYSIS OF INTEGRAL RELATIONS

With the chosen value of $T_1 = 300$ K in Fig. 4a in semilogarithmic coordinates, graphs, calculated using the integral relationship (8) and illustrating the relation of the value U_1 with the temperature T_0 on the ideally insulated surface of the dielectric layer of various polymeric materials are given (for polipiromellitimide, polyethylene terephthalate and polychlorotrifluoroethylene the temperature dependence of the thermal conductivity $\lambda(T)$ (see Fig. 1) extrapolated to the value $T = 400$ K). For each of the five considered materials at $T > 400$ K, the corresponding value U_1 can be assumed constant. In the case of polytetrafluoroethylene (position 5), this assumption is valid even at $T > 350$ K. It should be noted that with the decrease in the temperature T_1 of the cooled surface of the dielectric layer, the values U_1 increases for all the polymeric considered materials (Fig. 4b).

Choosing the same values for $T_1 = 300$ K and $T_0 = 400$ K for all the polymer considered materials, let us compare the temperature distributions in the dielectric layer. In Fig. 5 graphs of the dependences $T(\zeta)$ calculated using the integral relation (10) are given. In this case, for each of the materials, the above-mentioned range of variation with temperature of the thermal conductivity coefficient is taken into account. But for all the considered materials, the difference in the temperature distributions corresponding to the two limiting dependences $\lambda(T)$ (see Fig. 1) does not exceed the value $0.005(T_0 - T_1)$ at $\zeta < 0.999$. Therefore, these distributions pertaining to the fixed material are shown in Fig. 5 with one common schedule.

The distribution of temperature along the thickness of the layer of polytetrafluoroethylene is most uneven, which is associated with the largest change in the accepted temperature range of the resistivity (according to Fig. 3, by almost four orders). For polycarbonate, the electrical resistance changes only by a factor of 10, which leads to a more gently varying temperature over the thickness of the layer made of this material.

The calculated dependences $T(\zeta)$ make it possible to find the distribution over the thickness of the dielectric layer of the absolute value of the electric field strength. Using formulas (3) and (9), we write

$$E_*(\zeta) = \frac{E(\zeta)h}{U_1(T_0)} = \frac{jh}{U_1(T_0)} \rho(T(\zeta)) = \frac{I(T_0)}{U(T_0)} \rho(T(\zeta)).$$

The value $E_* = 1$ corresponds to the homogeneous distribution $E(\zeta)$. In Fig. 6, graphs of the dependence for all the polymer materials under consideration at the chosen values of $T_1 = 300$ K and $T_0 = 400$ K are presented in semilogarithmic coordinates. The most non-uniform distribution of the electric field strength arises in the layer of polytetrafluoroethylene: the lower boundary of the thermal conductivity coefficient corresponds to the values of $E_*(0) \approx 0.0050$ and $E_*(1) \approx 250.4$ (curve 5 with dark circles), and its upper boundary is $E_*(0) \approx 0.0052$ and $E_*(1) \approx 262.2$ (curve 5 with light circles). The main reason for this unevenness is the significant change in the electrical resistivity noted above for this material in this temperature range.

The impact on the distribution of the electric field strength changes of the dependence of thermal conductivity on temperature is most significant for the polychlorotrifluoroethylene

layer: for the lower border $E_*(0) \approx 0.0709$ and $E_*(1) \approx 17.80$, and for the upper boundary - $E_*(0) \approx 0.0786$ and $E_*(1) \approx 19.7$.

In the case of the other three polymeric materials this effect is insignificant and for each of them the dependences are shown in Fig. 6 one common schedule. The smallest change in the electric field strength is in the polycarbonate layer: $E_*(0) \approx 0.326$ and $E_*(1) \approx 3.27$, which is also associated with a relatively small change in the electrical resistivity in the temperature range under consideration.

5 CONCLUSION

On the basis of the formulated nonlinear mathematical model of steady-state thermal conductivity in a plane dielectric layer under the influence of a constant difference of electric potentials, integral relations are constructed. These relations describe the distribution of the electric field strength and temperature in this layer. Integral relations take into account the change of the electrical resistance and its thermal conductivity with temperature and are used to establish the relations between the highest values of temperature in the dielectric layer and the difference of electric potentials on the surfaces of this layer. For five different polymeric materials with known thermoelectric characteristics, a quantitative analysis of this relationship has been carried out and the temperature and electric field distributions over the thickness of the layer have been calculated. When comparing the results of this analysis, it was found that the highest values of the allowable working temperature and the limiting difference of electric potentials can be realized when polytetrafluoroethylene is used as a dielectric. The obtained results allow to make a comparative analysis of the expediency of using a specific polymer material as a dielectric in the designed electrical and radio engineering devices.

Acknowledgements: The research was supported by Ministry of Education and Science of the Russian Federation [grant nos. 9.2422.2017 and 9.7784.2017] and by the Russian Foundation for Basic Research projects No. 18-38-00618.

REFERENCES

- [1] R.E. Schramm, A.F. Clark, R.P. Reed, *A Compilation and Evaluation of Mechanical, Thermal and Electrical Properties of Selected Polymers*, Boulder (Colorado, US): National Bureau of Standards, (1973).
- [2] B.I. Sazhin, A.M. Lobanov, O.S. Romanovskaya, M.P. Ejdelnant, S.N. Koykov, *Elektricheskie svoystva polimerov*, L.: Himiya, (1977).
- [3] T. Blythe, D. Bloor, *Electrical Properties of Polimers, 2 edition*. Cambridge University Press, (2008).
- [4] L.N. Novichenok, Z.P. Shul'man, *Teplofizicheskie svoystva polimerov*, Minsk: Nauka i tehnika, (1971).
- [5] J.E. Mark (ed.), *Physical Properties of Polymers*, Springer, (2007).
- [6] G.A. Vorob'ev, Ju.P. Poholkov, Ju.D. Korolev, V.I. Merkulov, *Fizika dijelektrikov (oblast' sil'nyh polej)*, Tomsk: Izd-vo TPU, (2003).
- [7] J. Bailey (Ed.), *Properties and Behavior of Polymers. 2 Volume Set*, Wiley & Sons, Inc., Hoboken, New Jersey, (2011).
- [8] V.I. Mazhukin, A.V. Shapranov, A. A. Samokhin, A.Y. Ivochkin, "Mathematical modeling of non-equilibrium phase transition in rapidly heated thin liquid film", *Mathematica Montisnigri*, **27**, 65-90 (2013).

- [9] V.S. Zarubin, *Modelirovanie*. M.: Izdatel'skij centr Akademiya, (2013).
- [10] O.B. Feodoritova, N.D. Novikova, V.T. Zhukov, "Multigrid method for diffusion equations based on adaptive smoothing", *Matematika Montisnigri*, **36**, 14-26 (2016).
- [11] V.S. Zarubin, G.N. Kuvyrkin, I.Yu. Savel'eva, "Variacionny`j variant modeli teplovogo proboya sloya tverdogo die`lektrika pri postoyannom napryazhenii", *Radiooptika. MGTU im. N.E. Baumana. E`lektronny`j zhurnal*, **5**, 38-50, (2016).
- [12] M.P. Galanin, V.T. Zhukov, N.V. Klushnev, V.V. Lukin, A.S. Rodin, "On the solving of conjugate problems for definition of parameters of wrap and heat exchange between elements of constructions". *Matematika Montisnigri*, **39**, 5-16 (2017).
- [13] V.S. Zarubin, *Inzhenernye metody resheniya zadach teploprovodnosti*, M.: E`nergoatomizdat, (1983).
- [14] D.L. Rule, D.R. Smith, L.L. Sparks, *Thermal Conductivity of a Polyimide Film between 4.2 and 300 K, with Alumina Particles as Filler*, Boulder (Colorado, US): National Institute of Standards and Technology, (1990).
- [15] V.C. Bede, E.V. Samardukov, N.Yu. Gasteva, P.M. Schchonoba, "Effect of crystallinity on thermophysical properties of polytetrafluoroethylene", *Teplo Massoperenos*, **7**, 452–456, (1968).
- [16] V.S. Zarubin, G.N. Kuvyrkin, *Matematicheskie modeli mexaniki i e`lektrodinamiki sploshnoj sredy`*, M.: MGTU im. N.E. Baumana., (2008).

Received March 1, 2018