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HYDRODYNAMIC CHARACTERISTICS OF WEAKLY CONDUCTIVE LIQUID MEDIA IN THE NON-UNIFORM ELECTRIC FIELD

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Summary. Theoretical model of the ions formation in a liquid dielectric and flows caused by high electric field is proposed. The three-dimensional system of macroscopic pre-breakdown electro-hydrodynamic equations is written. The influence of electric field on the molecule dissociation rate is taken into account. The system includes the Poison equation for electric field potential, equation of ion formation and the Navier–Stokes equations with the electric force. Author's steady analytical electrodynamic solution of these equations for the electric field distribution and potential of spherical high voltage capacitor with liquid transformer oil type dielectric is described. Analytical non-stationary and numerical steady solutions for velocity distributions in liquid dielectric flows are obtained.

1 INTRODUCTION

Deviations from the Ohm's law for slightly ionized solid media in pre-breakdown uniform electric fields were experimentally discovered by Poole about 100 years ago [1]. In a weakly conductive liquid media the same experimental effect was obtained by M. Wien about 10 years later [2, 3].

Theoretically this exponential effect for considerable number of media was explained by Frenkel for solid dielectrics and by Onsager [3] for liquid weak electrolytes and for weakly conductive liquid dielectrics. The space charge and electro-hydrodynamic (EHD) flows have been observed in these dielectrics at the pre-breakdown conditions [4].

The space charge formation, according to [5], occurs in the pre-breakdown fields until all the EHD characteristics become steady. Herein, steady conduction can be as unipolar (corona-discharge type), as quasi-neutral (plasma or electrolyte type). The last was considered early in [5] and in the present work.

The pre-breakdown current–voltage theoretical and experimental characteristics of considerable media in non-uniform electric fields are described by us in [6]. Purpose of present work is researching of the electro-hydrodynamic flows, caused by these high non-uniform electric fields. These intense flows are observed in transformer oil type liquids [7, 8] with complex molecular structure [9]. The hydrodynamic transfer of high voltage space charge, appeared in considerable liquids, is described in [10]. In the review [8] the surface high voltage electrode effects influence on considerable pre-breakdown electro-hydrodynamic flows is researched. This

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influence must be taken into account at plan high voltage electrodes. When applied high voltage field is non-uniform, the volume effects influence on considerable flows is dominated.

The transport properties of charged and neutral components of weakly conducting media under consideration are investigated in [11,12]. In [11], using the quantum formula Kubo, a new method for calculating the conductivity of dense media has been developed. The calculations were carried out using the Monte-Carlo method as well as the molecular dynamics method. The Wigner–Liouville equation was used. The graphs of dependence of the studied media electrical conductivity on the initial parameters and their plasma non-ideality are presented. In [12], the thermal conductivity of the silicon electron gas is simulated, using the corresponding Fermi–Dirac distributions. Coefficients of thermal conductivity are obtained, and their dependence on temperature is presented graphically.

The transfer processes in the considered irreversible thermodynamics are used in [13]. The approach of the continuous medium and the local thermodynamic equilibrium are used. The transfer equations are derived with recommendations for their use in the analysis of thermal conductivity, electrical conductivity and diffusion. The formation of mixtures of liquids is used when applying a simple mathematical model from the equations of irreversible thermodynamics. The Navier–Stokes and heat inflow equations are solved. The formation of droplets during the evaporation of a liquid is substantiated [14].

In [15], the equations of a low-density plasma magnetosphere are written in a strong magnetic field. The Vlasov equation of kinetic is used. The equation of the magnetohydrodynamic mixture motion is derived. This method is used to study the evolution of the plasma under consideration.

2 THEORETICAL MODEL OF ELECTRO-HYDRODYNAMIC PROCESSES IN THE LIQUID INSULATOR

The following inequalities are valid for bulk charge concentrations $n_{\pm}$, impurities concentrations n_{p} and neutrals n_{a} concentrations in a low-conducting medium:

$$n_{\pm} \ll n_{\mathrm{a}}, \qquad n_{\mathrm{p}} \ll n_{\mathrm{a}}. \tag{1}$$

The above relations can be considered as a condition for a weak ionization (dissociation) or low conductivity in the considered medium. The rates of volume ionization (or dissociation) and recombination are supposed to be known as the thermodynamics functions of the above mentioned concentrations, temperature and field intensity $|\mathbf{E}|$ (up to the breakdown values). They can be represented in the following form:

$$W_{\mathrm{i}} = W_{\mathrm{i}}(n_{\mathrm{a}}, n_{\mathrm{p}}, T, |\mathbf{E}|) = W_{\mathrm{i}}(n_{\mathrm{a}}, n_{\mathrm{p}}, T, 0) f(|\mathbf{E}|). \tag{2}$$

Here, T is the absolute temperature; $|\mathbf{E}|$ is the intensity of the electric field. The rate of the ions recombination is

$$\begin{cases} W_{\mathrm{r}} = K_{\mathrm{r}} n_+ n_-, \\ K_{\mathrm{r}} = \dfrac{(b_+ + b_-)\,|e|Z}{\varepsilon\varepsilon_0}; \end{cases} \tag{3}$$

W_{i} denotes the rate of ionization (dissociation); W_{r} is the rate of ion recombination; Z is the valence; ε is the dielectric constant of liquid insulator; b_+ and b_- are the mobilities of the corresponding ions. The expression for K_{r} (recombination constant) was obtained by Langevin in 1903 and by Onsager in 1934 for particular case of weak electrolytes. This is the two-particle ion–ion recombination, when the energy excess is absorbed by a medium.

Onsager had shown weak influence of sub-breakdown electrical fields on K_{r} when direct and reverse processes are ionization and recombination correspondingly. But in the case when these processes are dissociation and reverse dissociation the influence of $\mathbf{E}$ on K_{r} can not be neglected. Besides weak electrolytes, the Langevin's formula is valid for dense gases with chemical reactions, where ionic conductivity is much greater than electronic one. Corresponding conductivity occurs due to various processes.

Among them, there are the neutral molecules (with ionic chemical bonds) dissociation and the neutral molecules (with covalent chemical bonds) ionization. The latter goes through the electron ionization from some neutral particle with relatively low ionization potential and attaches it to another with relatively high electron affinity.

For the ion diffusion coefficient, we have used more known Einstein–Nernst relation:

$$ZD_{\pm} = \frac{k_{\mathrm{B}} T b_{\pm}}{|e|}, \tag{4}$$

where k_{B} is the Boltzmann constant, T is supposed to be a constant and equals to 300 K, because the Joule heat is small for the pre-breakdown phenomena.

Function $f(\mathbf{E})$ describes the dependence of the ionization (dissociation) rate on the electric field intensity. The expression, describing it, was obtained by Frenkel for the solid dielectric and generalized by Ostroumov in [4] for the case when $Z > 1$ with ion chemical bonds in molecules. It was used in the present research, and has the form

$$f(\mathbf{E}) = \exp\left(\beta |\mathbf{E}|^{1/2}\right), \qquad \beta = \frac{|Ze|^{3/2}}{\sqrt{\pi\varepsilon\varepsilon_0} k_{\mathrm{B}} T}, \qquad Z = 2, \qquad \varepsilon = 2.5. \tag{5}$$

The value of valence Z for dissociating molecules of transformer oil was taken from [9], and the dielectric constant of transformer oil ε was taken from [4]. This function f of the high field shock ionization rate is not well known for gases.

The equations, describing creation and annihilation of the space charge, high voltage conductivity and the electric field distribution can be written as [5]

$$\begin{cases} \dfrac{\partial q}{\partial t} + (\mathbf{V}, \nabla q) - \dfrac{k_B T b}{|Ze|}\Delta q + (\mathbf{E}, \nabla\sigma) = -\dfrac{q\sigma}{\varepsilon\varepsilon_0}, \qquad \Delta\phi = -\dfrac{q}{\varepsilon\varepsilon_0}, \\ \dfrac{\partial \sigma}{\partial t} + (\mathbf{V}, \nabla\sigma) + b^2(\mathbf{E}, \nabla q) - \dfrac{k_B T b \Delta\sigma}{|Ze|}\dfrac{\sigma}{Z} - \dfrac{\sigma_0^2}{\varepsilon\varepsilon_0}\exp\left(\beta|\mathbf{E}|^{1/2}\right) + \dfrac{\sigma^2}{\varepsilon\varepsilon_0} = 0, \end{cases} \tag{6}$$

where q is the bulk charge, $\mathbf{V}$ is the velocity vector, σ is the electrical conductivity.

The well-known hydrodynamics equations should be added to equations (6) to construct the closed system of equations. The first of them is the continuity equation for incompressible media

$$\operatorname{div}\mathbf{V} = 0. \tag{7}$$

We used the equation (7) to obtain electrodynamic equations (6). For our purpose it is enough to consider the law of momentum conservation without considering the law of energy conservation. The momentum conservation equation can be written as

$$\begin{cases} \rho\left(\dfrac{\partial u}{\partial t} + u\left(\dfrac{\partial u}{\partial x}\right) + v\left(\dfrac{\partial u}{\partial y}\right)\right) = -\dfrac{\partial p}{\partial x} + \mu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) + qE_x, \\ \rho\left(\dfrac{\partial v}{\partial t} + u\left(\dfrac{\partial v}{\partial x}\right) + v\left(\dfrac{\partial v}{\partial y}\right)\right) = -\dfrac{\partial p}{\partial y} + \mu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right) + qE_y. \end{cases} \tag{8}$$

Some properties of oil and the initial conditions for the system of equations (6)–(8) are

$$\begin{aligned} &\mu = 0.00016 \text{ Pa s}, \qquad \rho = 800 \text{ kg/m}^3, \qquad \sigma_0 = 10^{-13} \text{ Ohm}^{-1}\text{cm}^{-1}, \\ &P|_{t=0} = P_0 = 10^5 \text{ Pa}, \qquad q|_{t=0} = 0, \qquad V|_{t=0} = 0. \end{aligned} \tag{9}$$

Upon further calculation, the following boundary conditions were adopted: the condition of adhesion on the surface of the electrode-wire and flat electrodes was set:

$$\mathbf{V}|_{\partial\Delta} = 0,$$

where $\partial\Delta$ is the boundary of the electrodes.

At the remaining boundaries of the computational domain, the condition of free flow was set:

$$\left.\frac{\partial \mathbf{V}}{\partial \mathbf{n}}\right|_{\partial\Omega} = 0,$$

where $\mathbf{n}$ is the external unit normal to the boundary $\partial\Omega$ of the calculation domain Ω.

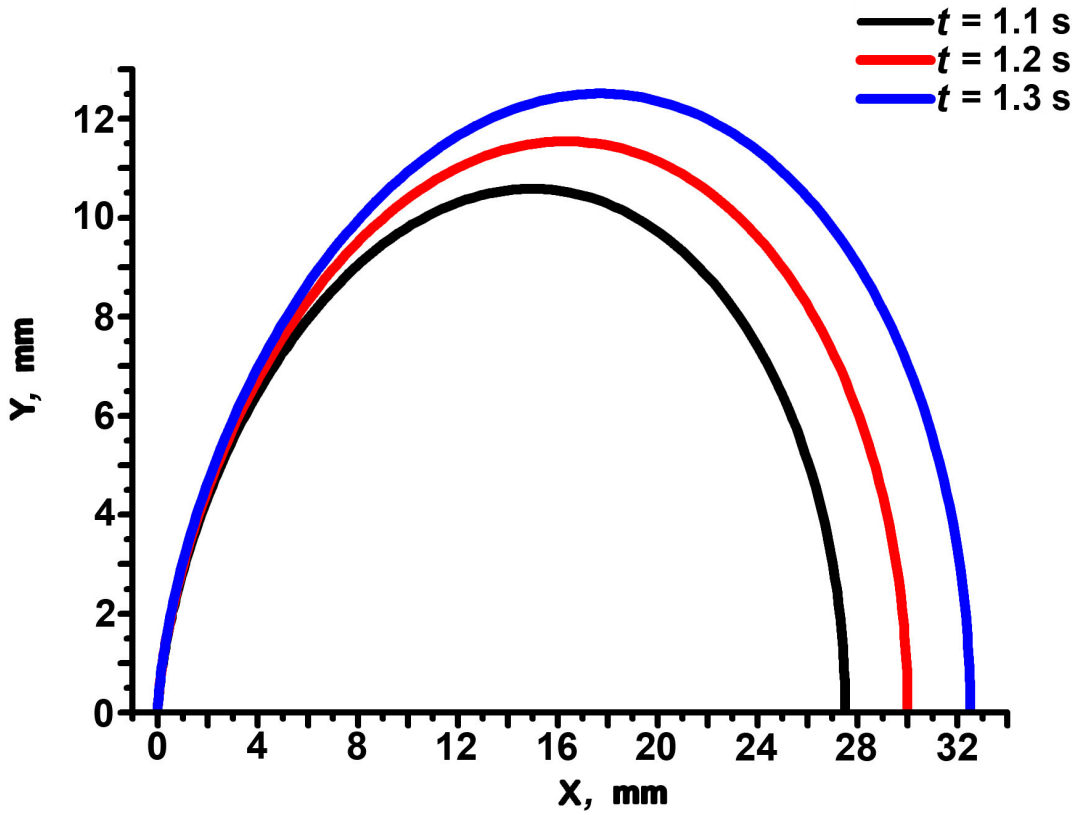

Figure 1: Graphics of the constant flow jets (12) for $U = 4$ kV and debit 0.1 L/s for different time moments.

The quasi-stationary analytical electrodynamic solution of equations (6) for the spherical symmetry electrical potential distribution in quasi-neutral medium can be obtained from charge conservation law (this well-known law can be obtained from equations (6) too). This solution looks like

$$\begin{gathered}\phi(r) = \left(\frac{I}{4\pi\sigma_0}\right)^{1/2}\left[\frac{8}{\beta} - \sqrt{\frac{\varepsilon\varepsilon_0}{\tau\sigma_0}}\left(|\mathbf{E}|^{1/2} + \frac{8}{\beta}\right)\right]\operatorname{sign}(\phi(r_0)), \\ |\mathbf{E}|\exp\left(\frac{\beta}{2}|\mathbf{E}|^{1/2}\right) = \frac{I}{4\pi\sigma_0 r^2}, \qquad |\phi(r_0)| = U.\end{gathered} \tag{10}$$

This quasi-neutral solution (10) is zero approaching of hydrodynamic space charge transfer differential operator series [10]

$$q = \sum_{i=0}^{\infty}(\tau v\nabla)^i\,\sigma\mathbf{E}\nabla\tau, \tag{11}$$

where τ is the charge relaxation time. Mathematical space of differential operators, obtained in [10], is not the Banach one. The quasi-exponential dependence as dependence [6] for volt-ampere pre-breakdown characteristics calculations of high voltage spherical capacitors can be

obtained from (10). These deviations were explained in [3] for plan capacitors early. The Laplas condition of pre-breakdown electric field, obtained in [7], can be also obtained from (10).

In contrary to the unipolar conduction solutions of our equations (6), the analytical formulae for pre-breakdown volt-ampere characteristics can be obtained analytically only for the case of cylindrical symmetry. This solution was obtained for slightly ionized gases in [16] early. The non-stationary hydrodynamic solution of equations (6)–(8) with the use of (9) and (10) for development of weakly conductive liquid jet flows from high voltage pinpoint electrode is

$$\Psi = \frac{\varepsilon\varepsilon_0 U^2 (t-\tau)\sin^2\theta}{32\pi\rho r}. \tag{12}$$

The graph of solution (12) is shown in figure 1. According to this graph, the jet flows are more developed, when time increases.

3 NUMERICAL CALCULATIONS

Numerical calculations of the isothermal two-dimensional system of equations (7)–(10) were carried out for the "wire above the plane" configuration of electrodes mentioned above. The need to determine the maximum speed of the fluid is important task due to the cooling problem of low-power non-pressure 10 kV transformers. We believe that the voltage inside such a transformer is proportional to the generated one and can reach value of 500 V and higher. The method applied and solving system of equations was described in detail earlier [17].

Calculations, mentioned in this paper, were carried out for the wire with square cross-section. This was done because the transverse dimension of the wire is much smaller than the linear dimensions of the calculation region. This region has to be not less than the distance from the wire to the flat electrode, and, in its turn, should be quite large to minimize the influence of boundary conditions on the calculation. As a compromise on the speed of calculation and flow details outside the region of wire electrode such a grid was chosen, in which the size of each side of the wire electrode equals to 8 computational cells.

When conducting full-scale experiments and in reality the cross section of wire electrode is a circle. There is a question: how reliable are the simulation results with the square-shape electrode and how much are they differ from the simulation results with a round-shape electrode? To determine this, four calculations were carried out: two on a coarse grid, figure 2(*a, b*), with a wire of square and "round" (as far as possible) cross-sections, and two on a finer grid with a similar sectional view, figure 2(*c, d*). The calculations were carried out until the flow was completely established.

Coarse grid parameters: 100×100 cells, cell size equals to 0.0005 m or 0.5 mm. Parameters of the finer grid: 500×500 cells, cell size equals to 0.0001 m or 0.1 mm.

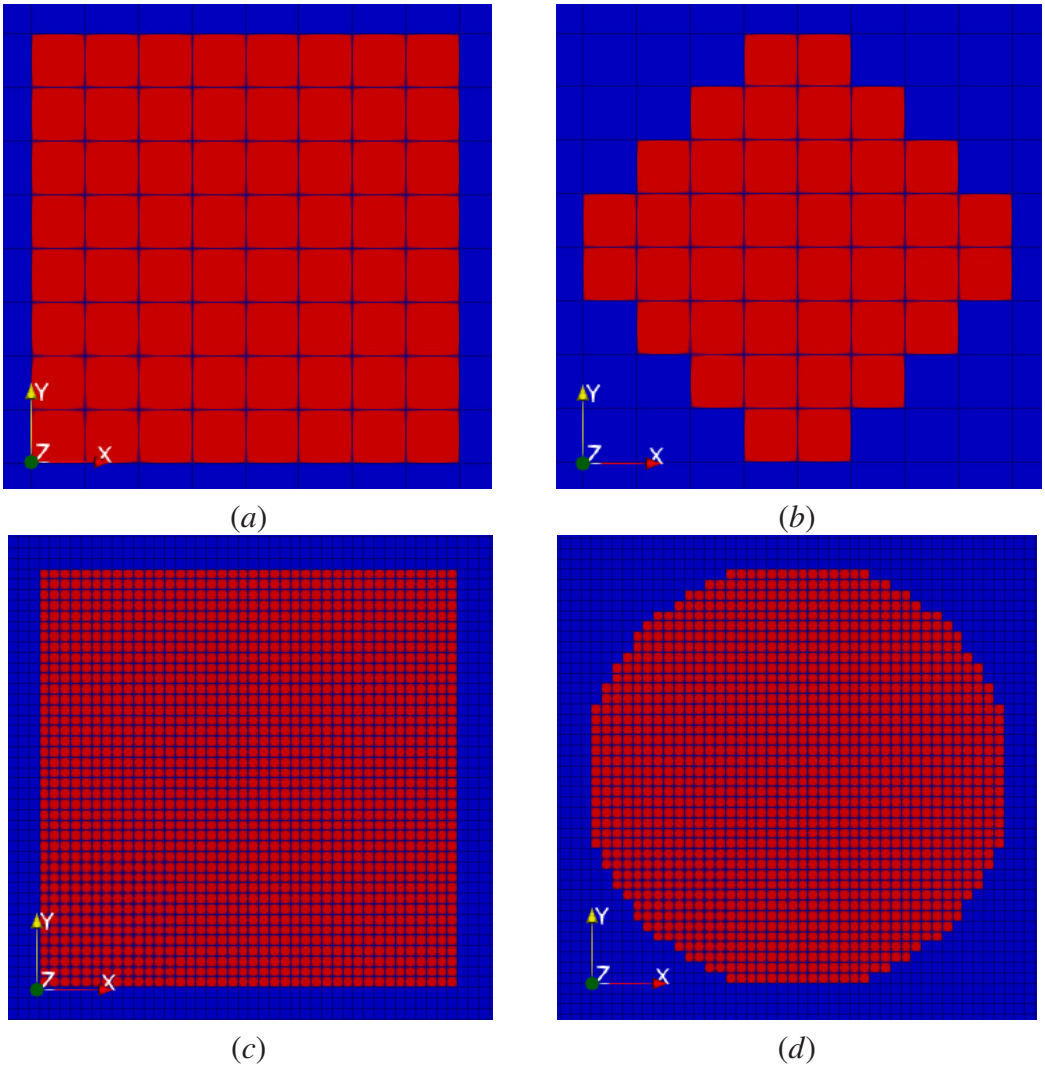

Figure 2: Square (*a, c*) and round (*b, d*) wire cross-sections on a coarse (*a, b*) and more detailed (*c, d*) grids.

Figure 3 demonstrates the axial velocities of fluid motion between the electrode-wire and the electrode-plane without taking into account the effect of the bulk charge. Voltage between the electrodes equals to 500 V. Inter-electrode distance equals to 2 cm. Different configuration of the wire cross-section and various fineness of the calculation grid are presented without taking into account the effect of space charge. One can see in figure 3 that the result has not changed qualitatively, but the shape of the electrode affects the quantitative result quite strongly. The maximum flow speed on the straight line connecting the electrodes increases with increasing fineness of the grid. In addition, when calculating on a fine grid in the region of a square-section electrode, the reverse flow is noticeable at a distance of 18 to 20 mm from the flat electrode, shown by the arrow in figures 3 and 4. Besides, the maximum of fluid velocity is greater if the shape of the wire cross-section is close to rounded.

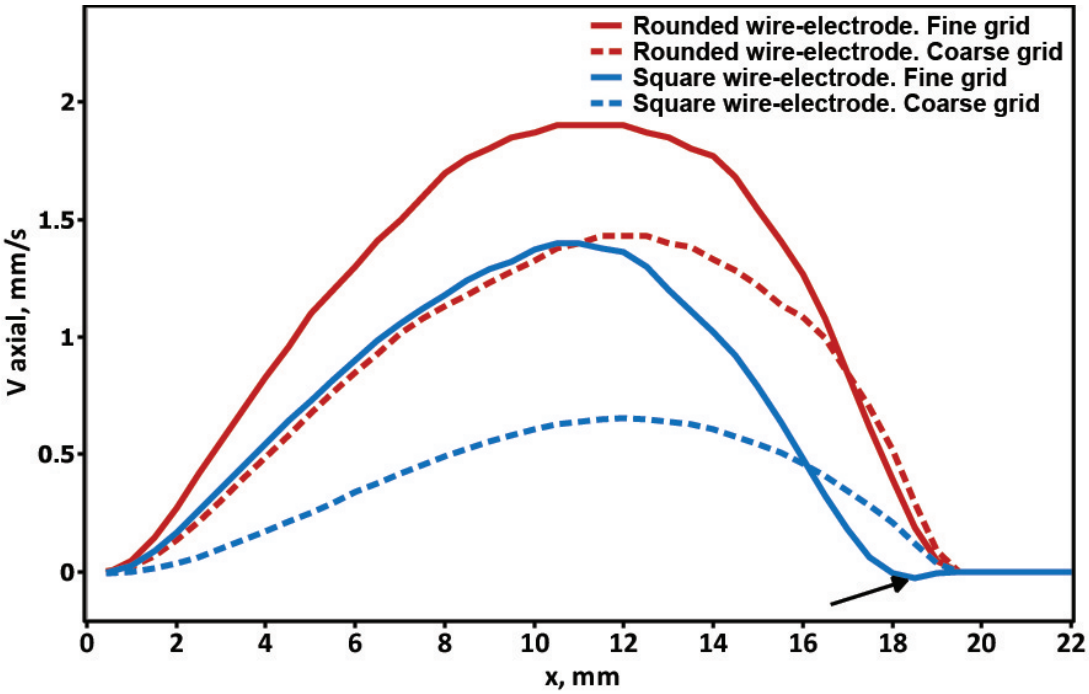

Figure 3: Distribution of the axial velocity between the electrodes in the electrode system "wire above the plane" in the axisymmetric electro-hydrodynamic flow of a weakly conducting liquid.

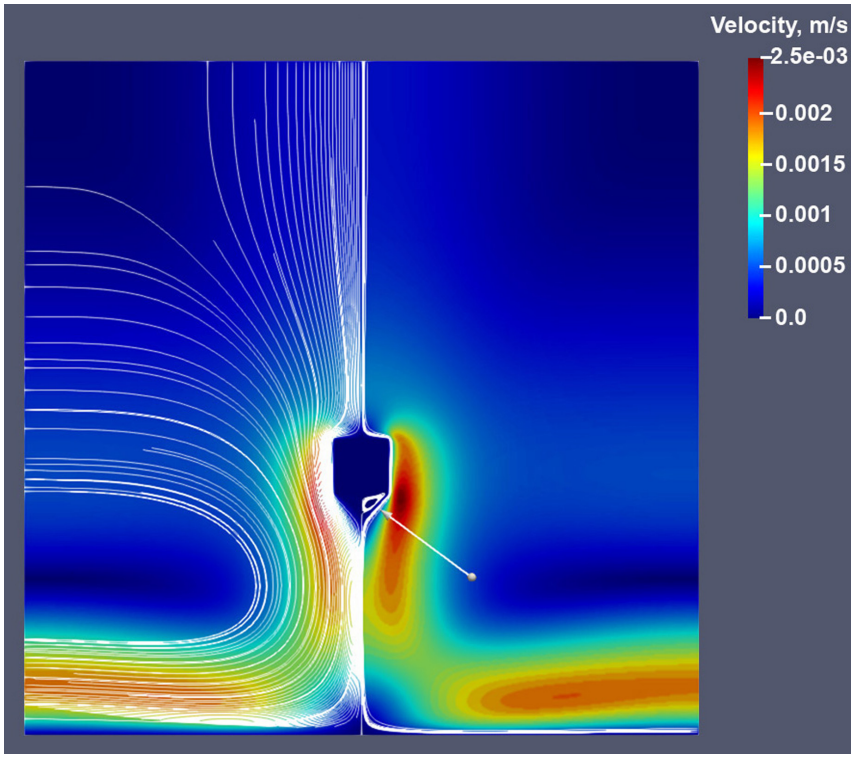

Figure 4: Streamlines and magnitude of the velocity for a weakly conducting liquid in an electric field without taking into account the effect of space charge. The cross section of the wire-electrode is square.

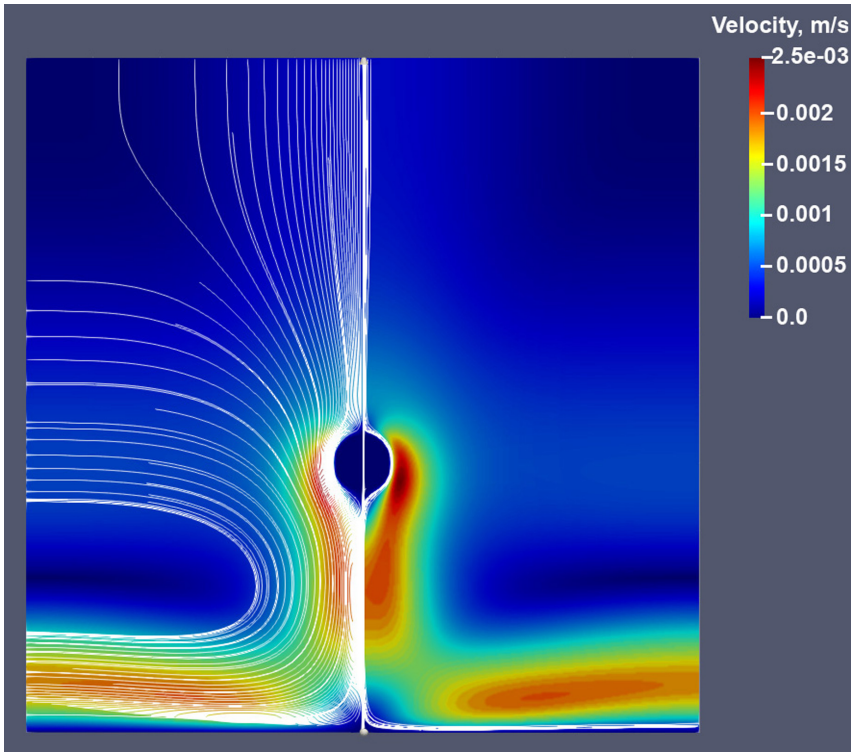

Figure 5: Same as in figure 4 but for the round cross section of the wire-electrode.

Figures 4 and 5 present comparative pictures of the motion of a weakly conducting liquid in the region of electrodes at a voltage of 500 V using a grid of 500×500 cells. It can be seen that in a case of the electrode with a circular cross section, the jets on both sides of the electrode are combined in a single jet at a distance of 10 mm from the flat electrode. In a case of the quadrate electrode, these two streams remain disconnected. It is also evident that the maximum fluid velocities are observed not in the central jet on the connection line of the two electrodes, but on both sides of the electrode-wire. And, as it was already mentioned earlier, there is a noticeable upward flow near the square electrode. This is not observed near the electrode with circular cross-section.

4 CONCLUSIONS

- For the pre-breakdown volt-ampere characteristics of transformer oil and liquid heptane, the squared current–voltage dependencies are obtained, and with decreasing electrode distance they become quasi-exponential. In addition, as the electrode distance decreases, the pre-breakdown current increases; this agrees with the results of the experiments.

- This work shows how fineness of the calculation grid and the cross-section of the wire electrode influence on the weakly conducting liquid flow in the inter-electrode space.

Flow structures and velocity distribution are obtained both for square and round shape of the wire electrode. The maximum axial velocity between the electrodes increases if the wire with the round cross-section is considered. Besides that, it is shown that this maximum axial velocity increases while using finer calculation grid. Further conclusions about what cross-section form should be chosen can be made after conducting relevant experiments.

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