MATHEMATICA MONTISNIGRI
Vol XXXI (2014)

A LOGICAL BURBAKIES MODEL PROGRAM FORMALIZATION OF MATHEMATICS

RADOSLAV MILOŠEVIĆ

University of East Sarajevo, Faculty of Philosophy Pale Aleksa Santic 1,
Republic of Srpska, Bosnia and Herzegovina
e-mail: radoslav_milosevic@yahoo.com

Summary. Gate mathematical theory requires special model axiomatic formalization of the mathematical theory. Classic formal mathematical theories, such as number theory, function theory, group theory, game theory, information theory, theory of surfaces, probability theory, and others have each individually own logical model construction. Each of these theories must satisfy the requirements uncontradictions, independence and thoroughness of his axiomatic system. Similarly in this paper we will present a logical model Burbakies program of formalization of mathematics. As an illustration of a solution that is technically in detail (especially for example. Introduction in how objects) quite different from the above theory of classical mathematics, with a tendency to be as adaptable to formalize a number of specific mathematical theory, we outline in brief strokes (tentative) program of formalization mathematics to Burbaki (Nicolas Bourbaki). Next, we will show how to Burbaki so by the program develops (or sketch) the formalization of some logical and mathematical theory: the logic of the statement, predicate logic, predicate logic with equality, set theory (etc., at least in principle).

1 INTRODUCTION

The philosophical orientation of Mathematics of the 20th century began with the reconstruction of all three programs of classical mathematics, where there are three program reimbursement basis of mathematics, namely: logicism (founder Bretnard Russell), formalism (founder of David Hilbert) and intuitionism (founder Brauer). As the criticism that followed the development of these programs created a new philosophical trends in contemporary mathematics that time, from which follows a series of founder intuitionistic - constructivist school, which belongs Brauer, Heitinga, Troelstra, Van Dalen (holansdski intiucionizam), then Markov, Kushner, Bishop, Majnes, Rajhman (American pragmatic constructivism) and Johanston, Staples (englesni sophisticated constructivism).

Along with these projects, the philosophical orientation in mathematics in France, appeared a group of mathematicians under the name Burbakisti who offered his program of reconstruction of mathematics.

Burbaki Nicholas alias group of French mathematicians of the younger generation, which was formed in 1937., and has set itself the task to realize the establishment's fundamental axiomatic mathematics, providing that the fundamental role of the structures. Revealed that they were among the founders of the group A. Kartan, A. Bale, Z. Djedone and K. Chevalier. Under the name Burbaki the beginning of 1939. Published scientific papers and a large series of monographs Elements des Mathematique.

2010 Mathematics Subject Classification: 03C55, 03C62, 03C65, 03C70.
Key words and Phrases: Logic, model, Burbaki, program, formalization.

Burbakisti, mathematicians gathered in a group or Burbaki about these groups, also those who accept and implement the notion of the group were in science or in teaching mathematics.

This paper presents a sketch of the logical model Burbakijevog program of formalization of mathematics.

2 LOGICAL INTERPRETATION OF PROGRAM BURBAKIES FORMALIZATION OF MATHEMATICS

We assume that the choice of the structure of a formalized mathematical theory there is a lot of freedom and that decisions about it are not only mathematically and logically motivated. As an illustration of a solution that is technically in detail (especially for example. Way of introducing the objects that correspond to bound variables), and is designed, among other things, with a tendency to be more adaptable to the formalization of a large number of specific mathematical theory, we outline in brief strokes (tentative) program of formalization of mathematics to Burbaki (Nicolas Bourbaki).

Signs and words ("assemblages"). Signs of a mathematical theory T are these: Logical 1 means: □ , □ , ¬ .

The letters 2, which are introduced as required; It will regularly be sensitive of the Latin alphabet, possibly with accents, for example: A, A', A", ...

Specific means 3, which depend on the considered theory. (Eg. for set theory to the = sign, □, □).

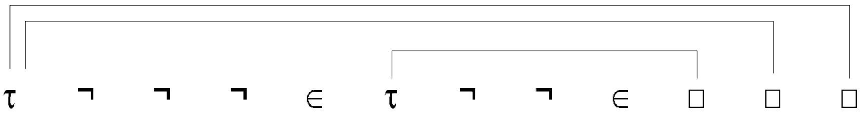
Word of the theory T strings of T are written next to each other, some characters (but not the letters) can be connected in pairs moves above characters that call-ups. For example: in set theory question.

Exclusive printing words, effective as they are, would lead quickly to virtually insurmountable difficulties (because of their often large lengths of tens of thousands of characters, and more), and therefore for them to introduce abbreviations, among other things, the words of ordinary speech - which, of course, not belong to the formal mathematics and theory are urgently needed. For example: □ stands for ¬ and O is an abbreviation for (empty set).

$$\tau \quad w \quad \neg \quad \in \quad \square \quad A' \quad \in \quad \square \quad A''$$

Some do mathematical (meta) theory, less theory contains rules that allow you to say certain words that are terms or relations theory, and other rules that allow you to say certain words that are theorems of the theory. Description of the rules (below) does not belong to itself (formal) theory; it is frequently used abbreviations and various substitute word or parts thereof, the meaning of which is usually clear from the context. Regularly, for brevity, will mean that these abbreviations words, instead of indicating them. (For example: if A, V word, AV will be the word that comes concatenating character of V, in order to come behind the signs of A, in the order they come).

Let A be a word, a letter h□h(A) will mark the word that gets this in words □A Connect to tie each h wherever you come in and □ to the left of A and then replace h wherever you come in and with □ (It □h(A) therefore does not contain h). Eg: □ h (xy□) meaning of the word.



Let A, V word, and the letter h. Word that result from replacing By changing h wherever comes with a V means the following: (V| h)A.

For example: with A ≡ □ xy = xx, V ≡ □ being (V | h) A ≡ □ □ □ □ = y.

If we want to point out one or two different letters, for example h and h, and in which can (but need not) coming to A, writing the Ax{A} or {x, y}. In such a case, for example A{V} is the same as (V|h)A; analogous to A{V, S} with simultaneous replacement of h with a V, with the S in A. The possibility of confusion to avoid the most common (in abbreviation) additional brackets. For example: replacing R□Q for N and M□N received word it shall be indicated with M□(R□Q).

Criteria substitutions. Formal mathematics contains only the explicit written word. Practically, however, are required (theoretically not required) criteria for certain manipulations with the words - of course, they belong to the proper meta. Thus, the criteria of substitution are obtained. Among them, for example:

CS 1. Let A, V words, h, h' case. If h' is not an A, then (V|H) ≡ A (V|h')(h'|h)A.

CS 2. Let A, V field, and h the (codes for) various letters. If x out of the V, then (V|u)□h(A)≡□h(A'), where A' is ≡ (V|u)A.

Formative structures. Among the specific character of each is called relational, others substantive. Each specific sign associated with its weight; it is an integer, usually 2.

The word is called the first kind it starts with □ or substantive character, or if it consists of only one letter; Otherwise called the other species.

Formative design theory T is a sequence of words with this feature: for every word A series filled with one of the following characteristics:

a) A letter is.

b) In the series there in front of a word other types of V such that A ≡ ¬ V.

c) In the front row there are a word other types B, C such that A ≡ ¬ VS.

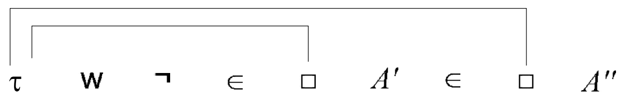
g) there is a row in front of the other types and the word V, so that for a letter Ahh≡ □(V).

d) There is a specific sign of the weight n8 benzyl and n the words of the first type A1, A2,

e) . . . an outside A, such that A is A1 A2 s ≡ . . , An.

Terme (or relations) of T are the words of the first kind (or other species) to come among the members of the formative structure of T. For example in set theory is □ relational character of weight 2, and the following series of formative structures:

A, A', A''□A, A', □A, A'', ¬ □ A, A', ¬ □ A, A' □ A, A'',



Its members are, for example: last, thermal theory.

Intuitive Talaso corresponding objects (the classical theory that should be formally "cover"), and the relations between relations such facilities. In particular, for example in g) if V is a proposition which expresses a property of the object h, then h□(V) corresponds to a"privileged" object that owns the property, if it exists, otherwise it represents an object about which nothing can be said.

Formative criteria allow the words you already know that the spa or relationships concluded for such a word. For example: let A be a relation (or term) of a theory T, h letter, a term T of T. Then (T| h)A relation (or term) of T.

Axioms. Setting its specific character and defines the thermal relations of a theory T. To conclude construction of T, should be :

1) Print a certain relation of T to be called explicit axioms of T; letters that come in explicit axioms are called constants of T.
2) Set the one or more rules which are referred to the scheme of the T and which must have the properties of:
   a) the application of such a rule R gives a relation of T;
   b) If T is a terma of T, the letter h, R relation of T is constructed using the scheme

R, then the relation (T| h)R can be obtained by applying R.

Any relation obtained by applying a scheme of T is called the implicit axiom of T.

Intuitively, an axiom is any obvious claims, any hypothesis that is accepted to be of consequence, they withdrew; Constants are some specific objects, which are assumed to possess properties making them explicit axioms express. On the contrary, if the letter h is not constant, it is a completely indeterminate object; If an axiom assumes that the object x has a particular characteristic, such is necessarily implicit axiom, so this feature then truth of whatever object T.

Demonstrations. Each demonstration text of a theory T contains:

1. Auxiliary formatted structure of some relation and the baths of T.

2. Some of the demonstration of T, ie. a set of relations of T to come in the formative auxiliary structure, such that for every relation R of the array filled with at least one of these conditions:

a1) R is an explicit axiom of T.

a2) R result of applying some diagrams of T on thermal or relationships that come in the auxiliary formative structure.

b) In a series of R are in front of the relation S, T, such that T S ≡ ¬ R.

Theorem of T is a relation of T that comes in a demonstrations of T.

The notion of a theorem relative to the state of the construction discussed theories; a relation of T becomes a theorem when he managed to get involved in a demonstration of T.

Let R be a relation of T, h letter, a term T of T; if (T|h)R theorem of T, says that T(a) solution of R in T.

For a theory T is said to be contradictory if (when) found a relation R of T for which it was shown that the R and ¬ R theorem of T.

Deductive criteria are mathematical rules that allow shortening the conclusion of the demonstration, for example:

S1. Let A theorem of a theory T, T theorem of T, and the letter h is not a constant of T. Then (T|h)A theorem of T.

Comparison theory. The theory T' is called stronger than theory T if you are all trademarks of T and signs of T', all explicit axioms of T theorem of T' and all the schemes of the T schema of T'.

S2. If the theory T' is stronger than the theory T, all theorems of T are theorems of T'.

3 INSTEAD OF CONCLUSION

Burbaki to set this program develops (or sketch) the formalization of some logical and mathematical theory: the logic of the statement, predicate logic, predicate logic with equality, set theory (etc. at least in principle) - in which is not discussed. At the end of the note, and that is a critical note on intuitionism in the preface Burbakijevog description of the formalization of mathematics in a review of that work in one of the world's leading mathematical journals of reference experienced (maybe a little pointedly, but perhaps justified) "opponent" Burbakijevog entire program of formalization of mathematics.

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Received March, 15 2014.